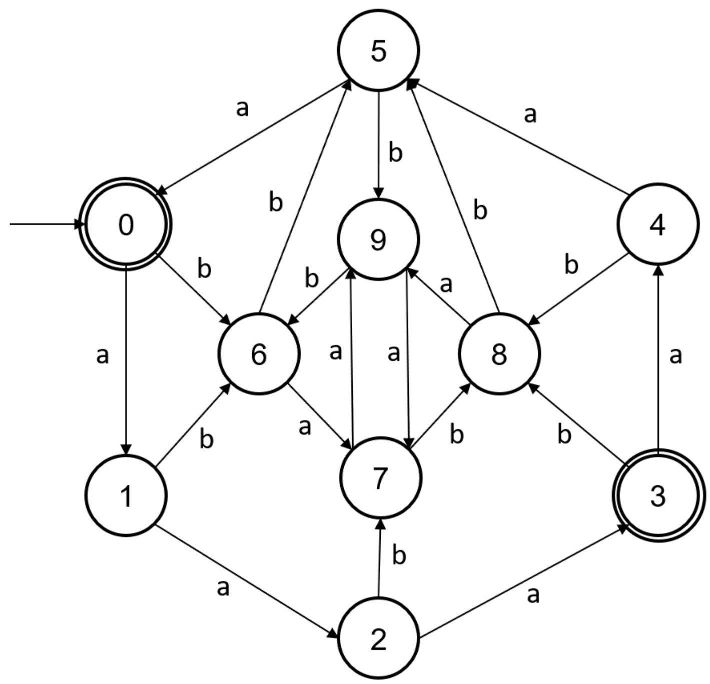
2FA3 - Assignment 2

Submitted via Avenue. Due March 12th, 11:59pm. **I will accept it late without penalty up until March 18th**. Note: I indicate what each question if worth by square brackets, i.e. [k].

1. Give regular expressions for the languages below [6]:
   (a) $L = \{a^n b^m \mid (n+m)\%2 = 0\}$
   (b) $L = \{w \mid w$ does not contain the substring: $aba\}$
   (c) $L = \{w \mid w$ has an even number of b's$\}$
2. Minimize the number of states in this DFA via quotient construction. Show all your steps. Specifically, in your table where you are "marking" nodes indicate which iteration you marked them on. [7]



3. Your friend is looking at the formal definition of the pumping lemma and they think something is wrong.

$$L \text{ is regular } \Rightarrow (\exists k \mid k > 0 : (\forall x, y, z \mid xyz \in L \wedge |y| > k : (\exists u, v, w \mid y = uvw \wedge v \neq \epsilon : (\forall i \mid i \geq 0 : xuv^i wz \in L))))$$

They understand the argument it is crafted around. That is, due to the fact that strings are arbitrarily long and a DFA has finite states there must be a segment of accepted strings which "loop" in the machine. However, they claim for the pumping lemma above to hold, L must be infinite, because if L was finite the argument about "looping" no longer holds. Therefore, the pumping lemma only holds when L is infinite.

You can see where your friend is coming from, but they are incorrect. Why? Be precise in your argument, that is, show how if L is finite, then

$$(\exists k \mid k > 0 : (\forall x, y, z \mid xyz \in L \wedge |y| > k : (\exists u, v, w \mid y = uvw \wedge v \neq \epsilon : (\forall i \mid i \geq 0 : xuv^i wz \in L))))$$

evaluates to true. [4]

Hint: If L is finite, there is a "longest string".

4. Using the Pumping Lemma, prove the following languages are not regular. Make your steps in the "game" and variable choices very clear for each question.[9]

   (a) $L = \{a^{nm}b^m a^n \mid n, m \geq 0\}$

   (b) $L = \{ww \mid w \in \Sigma^*\}$

   (c) $L = \{a^{k^3} \mid k \geq 0\}$