Real Time Systems and Control Applications

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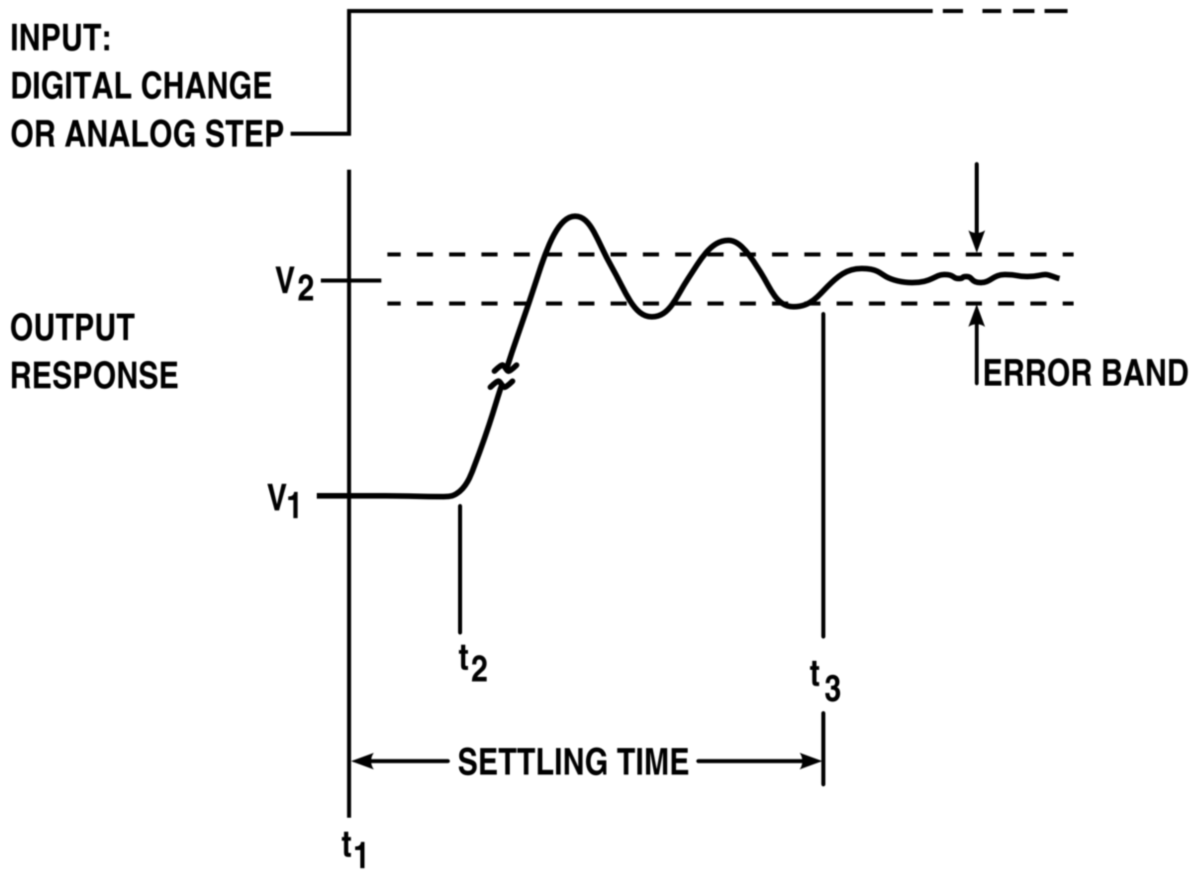
Control Systems

- What is a control system?
  - Desired output, desired performance with specified input
  - Performance: transient response, steady state error
- Types of Systems
  - Open Loop
  - Closed loop
  - Multi-loop



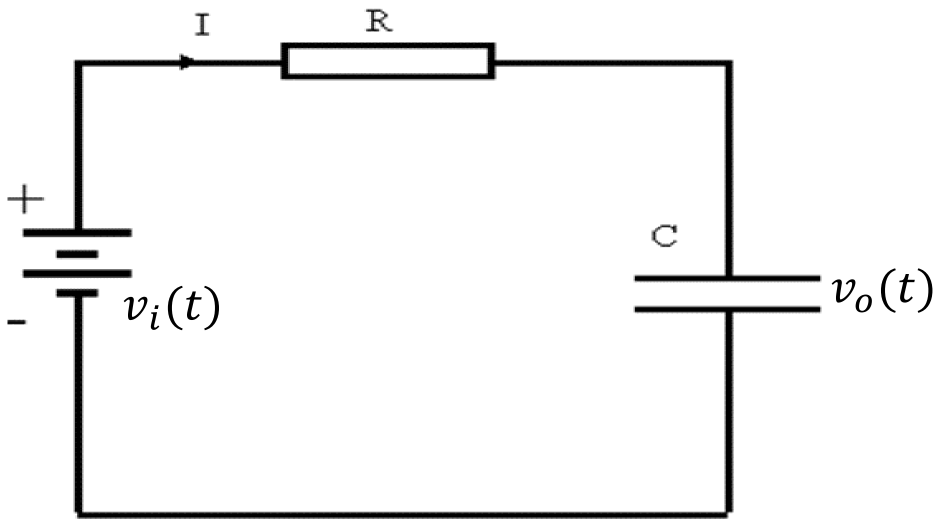
Time Domain v.s. Frequency Domain

- Actually, the input and output are in time domain, why we need Laplace Transform, and wish to investigate the system behavior in frequency domain?

- Consider a very simple circuit system:

$$RC\frac{dv_o(t)}{dt} + v_o(t) = v_i(t)$$

$$v_i(t) = 1$$



How To Solve The Ordinary Differential Equation?

- In time domain by solving differential equation, we have:

$$v_o(t) = 1 + k\, e^{-\frac{1}{RC}t}$$

Need initial condition of $v_o(0)$ to solve k. If $v_o(0)$ =0, k=-1.

Hence, $v_o(t) = 1 - e^{-\frac{1}{RC}t}$

- In frequency domain by Laplace Transform (LT), we have:

sRC$v_o(s) + v_o(s) = v_i(s)$ ➔ (sRC+1)$v_o(s)$=$v_i(s)$

Hence, $v_o(s) = \frac{\frac{1}{s}}{1+sRC} = \frac{1}{s+s^2RC}$

Inverse Laplace Transform: $L^{-1}\left(\frac{1}{s+s^2RC}\right) = 1 - e^{-\frac{1}{RC}t}$

Laplace Transform

Laplace transformation from the time domain to the frequency domain transforms differential equations into algebraic equations and convolution into multiplication.

Laplace Transform:

$$F(s) = \int_0^{\infty} f(t)\ e^{-st}\, dt$$

Inverse Laplace Transform:

$$f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s)\ e^{st}\, ds$$

Table of Laplace Transforms: http://tutorial.math.lamar.edu/pdf/Laplace_Table.pdf

Recall An Example

- Unit step function $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$
- $\mathrm{U}(s) = \mathcal{L}\{u(t)\} = \int_0^\infty u(t)\, e^{-st}\, dt = \int_0^\infty e^{-st}\, dt = -\frac{1}{s}\, e^{-st}|_0^\infty$

$$\mathrm{U}(s) = -\frac{1}{s}(0 - 1) = \frac{1}{s}$$

Note that Laplace transforms of various functions are found in tables. Similarly tables exist for inverse Laplace transforms.

Laplace Transform of Derivatives and Integrals

If $\mathcal{L}[f(t)] = F(s)$, then

$$\mathcal{L}[f'(t)] = sF(s) - f(0)$$

and

$$\mathcal{L}\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$$

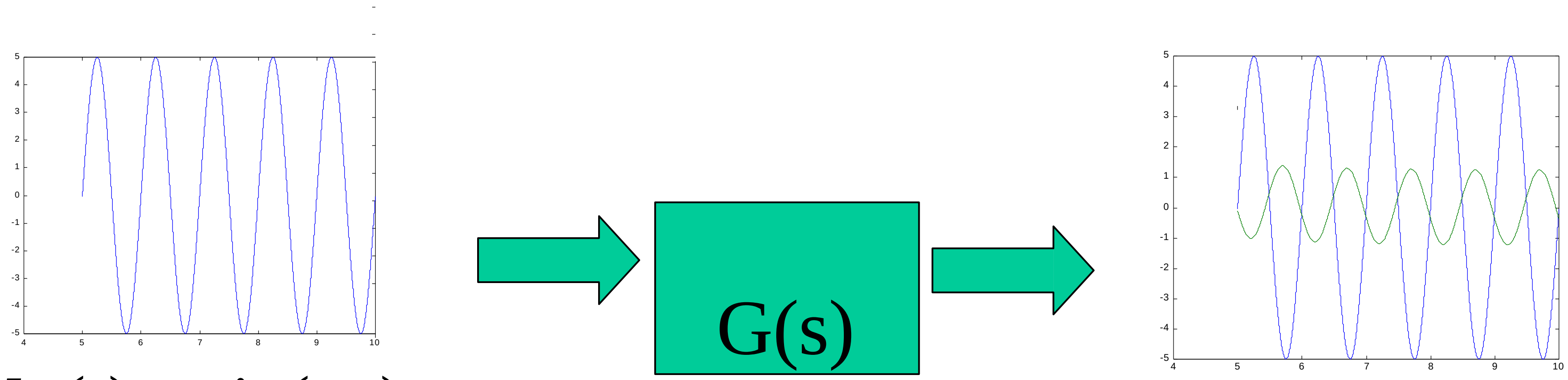
For higher order derivatives

$$\mathcal{L}[f''(t)] = s^2F(s) - sf(0) - f'(0)$$

A convenient method to transform differential equations to algebraic equations.

Frequency Response

For a sinusoidal input, the output of a linear system is also a sinusoidal. However, the output will have a **different magnitude** and will be subject to **a phase shift**.

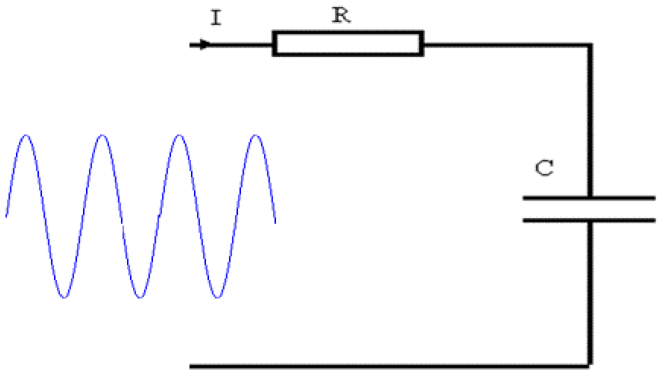


$V_{in}(t) = \sin(\omega t)$

$vel(t) = a_3 \sin(\omega t + \phi)$

Another Form of System Model in Frequency Domain

- Replace s with jw in transfer function $G(s)$ to get $G(jw)$
- $G(jw) = \frac{1}{1+jwRC}$



- So $|G(jw)| = \left|\frac{1}{1+jwRC}\right| = \frac{1}{\sqrt{1+(wRC)^2}}$

$$\angle G(jw) = \tan^{-1}(-wRC)$$

An expression for the gain where frequency is a variable. Allows calculation of gain at any frequency.

It is also possible to calculate the phase shift of the output relative to the phase angle of the input.

Why s Is Substituted With jw In Transfer Function?

Laplace transform:

$$G(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)\ e^{-st}\,dt$$

Fourier transform:

$$H(s) = \mathcal{F}\{f(t)\} = \int_0^\infty f(t)\ e^{-jwt}\,dt$$

The only difference between Laplace transform and Fourier transform is the variable substitution s=jw. Hence, $H(s) = G(s)|_{s=jw} = G(jw)$

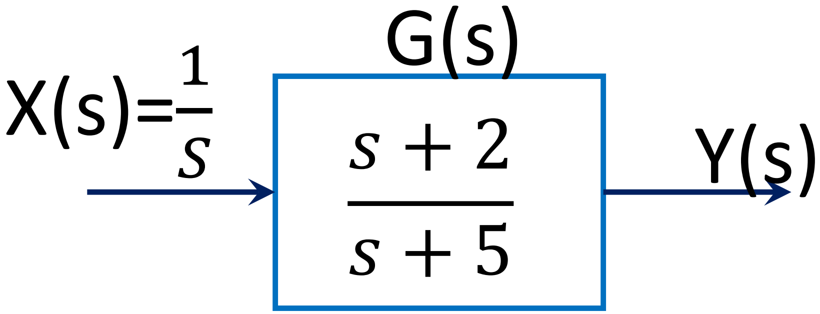
Time Response

- Why is time response important?
- Temperature control - how long it takes to reach a new steady state?
- Want 200 deg but it goes to 250 before settling down, will it be acceptable?
- How about cruise control at 100 km/h but actual speed varies between 80 and 120?

Next, let us examine the parameters that control the time behaviour of systems.

Consider First Order Systems

$$Y(s) = \frac{s+2}{s(s+5)} = \frac{{}^{2}/_{5}}{s} + \frac{{}^{3}/_{5}}{s+5}$$



$$y(t) = {}^{2}/_{5} + {}^{3}/_{5}\ e^{-5t}$$

Forced response Natural response

The output response of a system consists of:

(1) a natural (**transient**) response: ${}^{3}/_{5}\ e^{-5t}$

(2) a forced response (**steady state**) response: ${}^{2}/_{5}$

Poles and Zeros

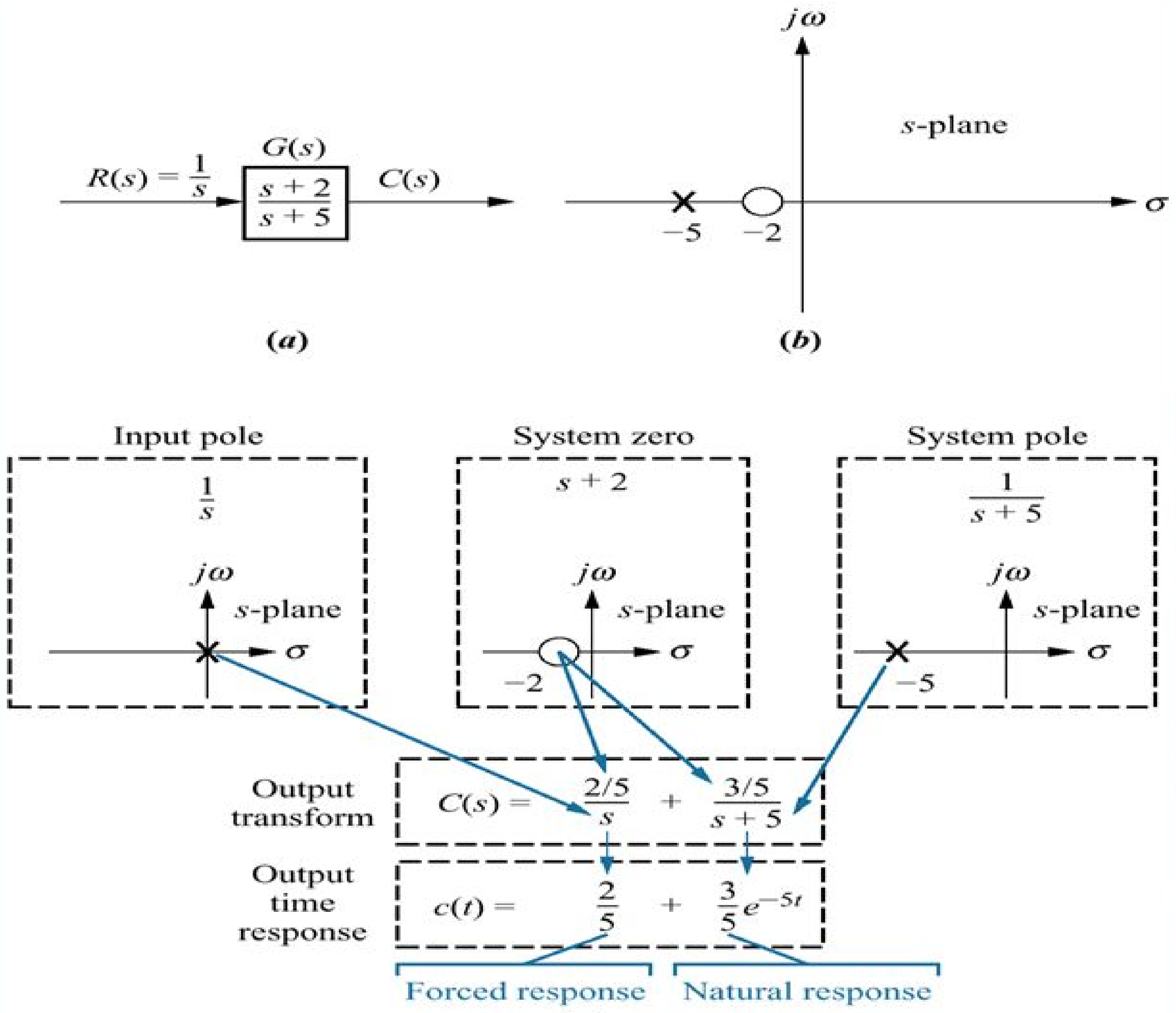
- In previous example: $Y(s) = \frac{s+2}{s(s+5)}$

s = 0, s = - 5 are poles and s = - 2 is the zero of the transfer function.

(1) A pole at origin generated a step function at the output.

(2) The pole at -5 generated transient response e^{-5t}. The further to the left a pole is on the negative real axis, the faster the exponential transient response will decay to zero.

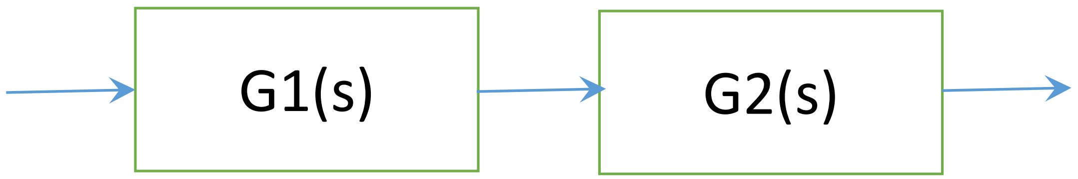
(3) The zeros and poles generate the amplitude for both the forced as well as the natural responses.

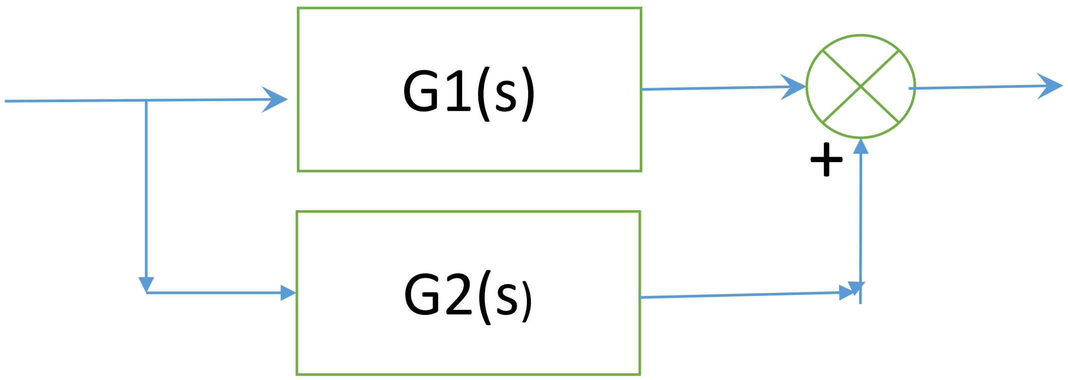


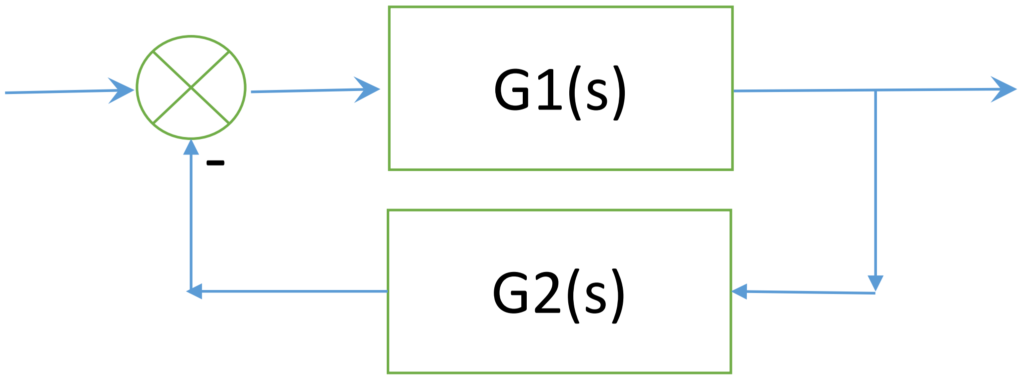
$R(s) = \frac{1}{s}$
$G(s)$
$\frac{s+2}{s+5}$
$C(s)$
(a)
$j\omega$
s-plane
σ
−5
−2
(b)
Input pole
$\frac{1}{s}$
System zero
$s + 2$
System pole
$\frac{1}{s+5}$
Output transform
$C(s) = \frac{2/5}{s} + \frac{3/5}{s+5}$
Output time response
$c(t) = \frac{2}{5} + \frac{3}{5}e^{-5t}$
Forced response
Natural response

More Reviews

Transfer Functions



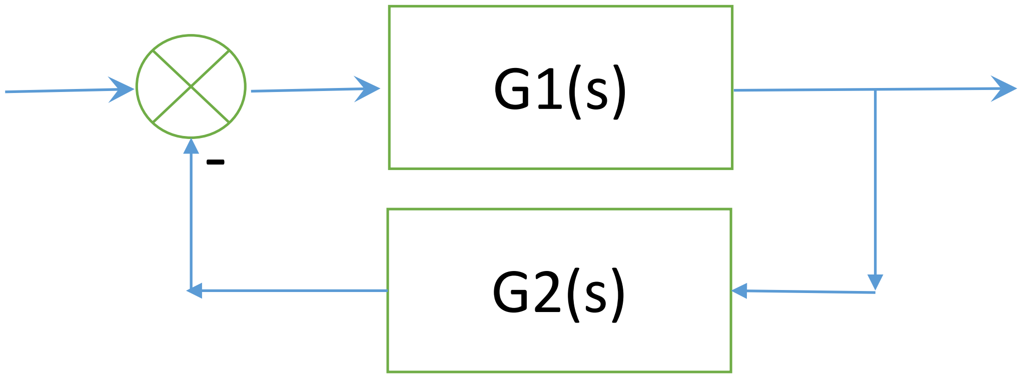




What are the system transfer functions?

Characteristic Equation of System

The **characteristic equation** is setting the denominator of the closed-loop transfer function to zero (0).



Characteristic equation of the system is

$$1+G1(s)G2(s)=0$$

Stability?

- Is this system stable?