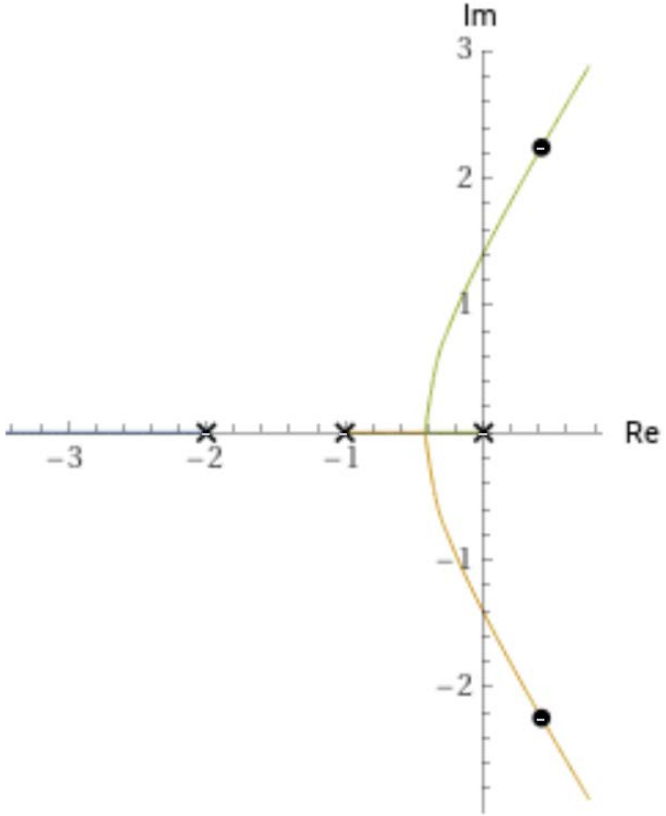
Real Time Systems and Control Applications

Contents

PID Controller

Stability Range of K?

- Consider an open loop transfer function G(s)= K/s(s+1)(s + 2). What K values make the closed loop system stable?



Need to get K so that the root locus intercept imaginary axis.

Consider the characteristic equation:
1+K/s(s+1)(s + 2)=0

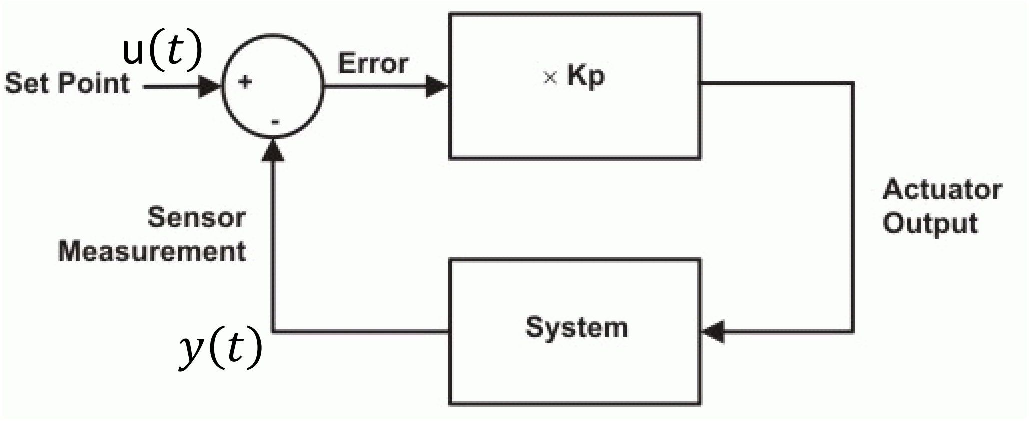
So $s^3 + 3s^2 + 2s + K = 0$

Assuming s = βj, solve K=6 when root locus intercepts the imagery axis, so the stability range of K is (0, 6).

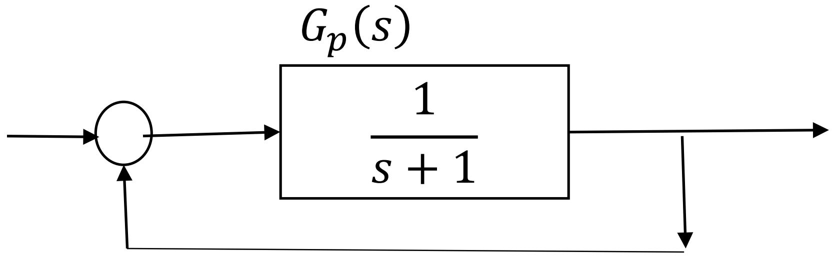
Proportional Control

Proportional control action is obtained when we multiply the error signal by a constant gain, i.e. our control input to the plant has a value proportional to the value of the error:

$$K_p e(t) = K_p[u(t) - y(t)]$$

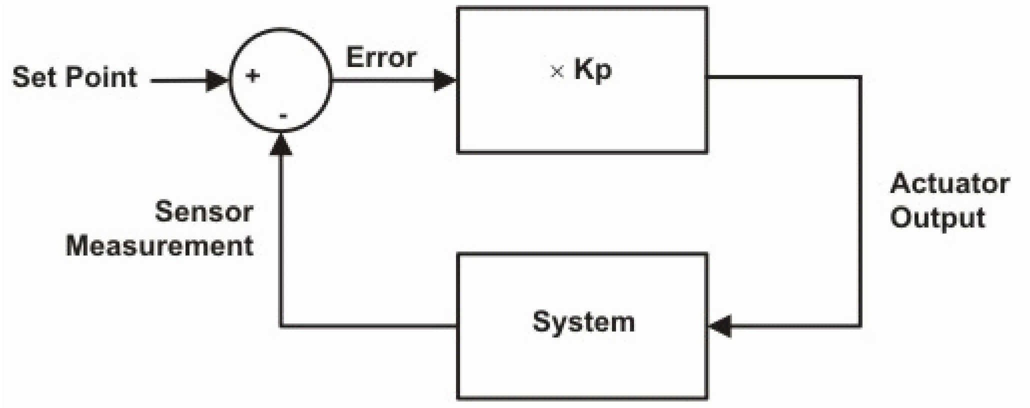


Example



- Closed loop transfer function is: $T(s) = \frac{G_p(s)}{1+G_p(s)} = \frac{1}{s+2}$
- Time constant: $\frac{1}{2}$ second
- The steady state output is: $y_{ss}(t) = \lim_{s\to 0} s\,T(s)\;u(s) = \frac{1}{2}$
- The steady state error is : $\frac{1}{2}$

Adding Proportional Control



- Closed loop Transfer Function is

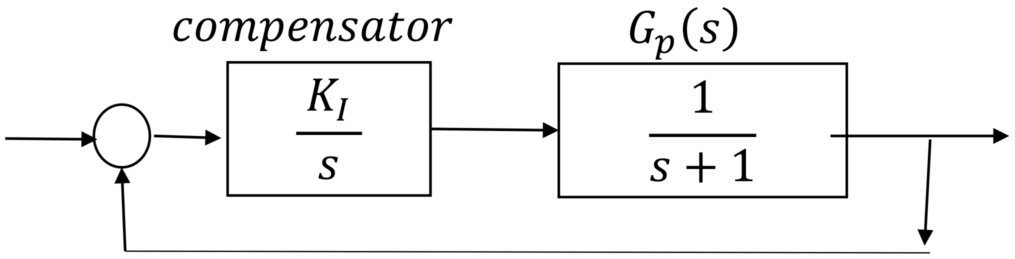
$$T(s) = \frac{K_p G_p}{1 + K_p G_p}$$

For $K_p = 9$ and $G_p = \frac{1}{s+1}$ ➔ $T(s) = \frac{9}{s+10}$, then the steady state output is:

$$Y(s) = \lim_{s \to 0}(s\, T(s)\, u(s)) = \lim_{s \to 0} \frac{9}{s + 10} = 0.9$$

The steady state error has been changed to 0.1, and $\tau = 0.1$

Integral Control



- The closed loop Transfer Function is

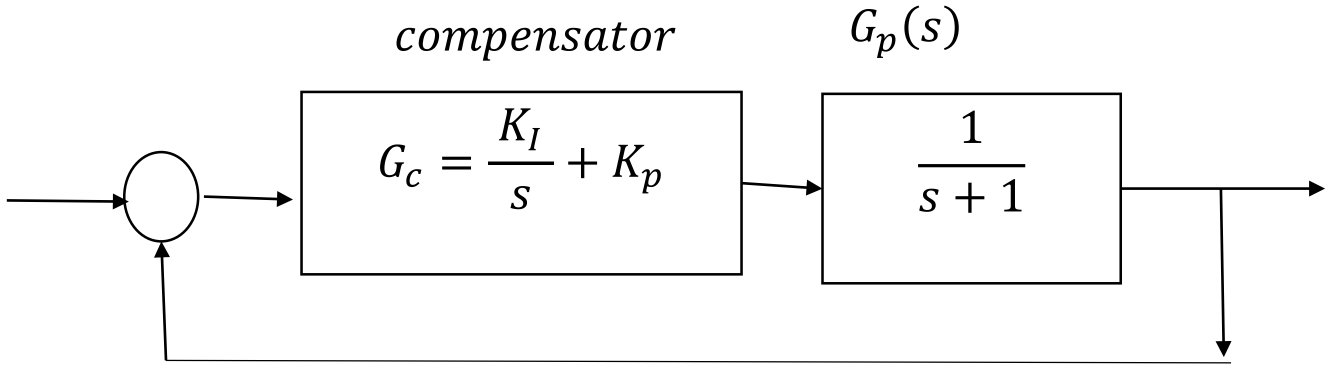
$$T(s) = \frac{G_c G_p}{1 + G_c G_p} = \frac{K_I}{s^2 + s + K_I}$$

The steady state output is

$$Y(s) = \lim_{s \to 0} (s\, T(s)\, u(s)) = \lim_{s \to 0} \frac{K_I}{s^2 + s + K_I} = 1$$

So the steady state error is 0.

Proportional-Integral (PI) Control

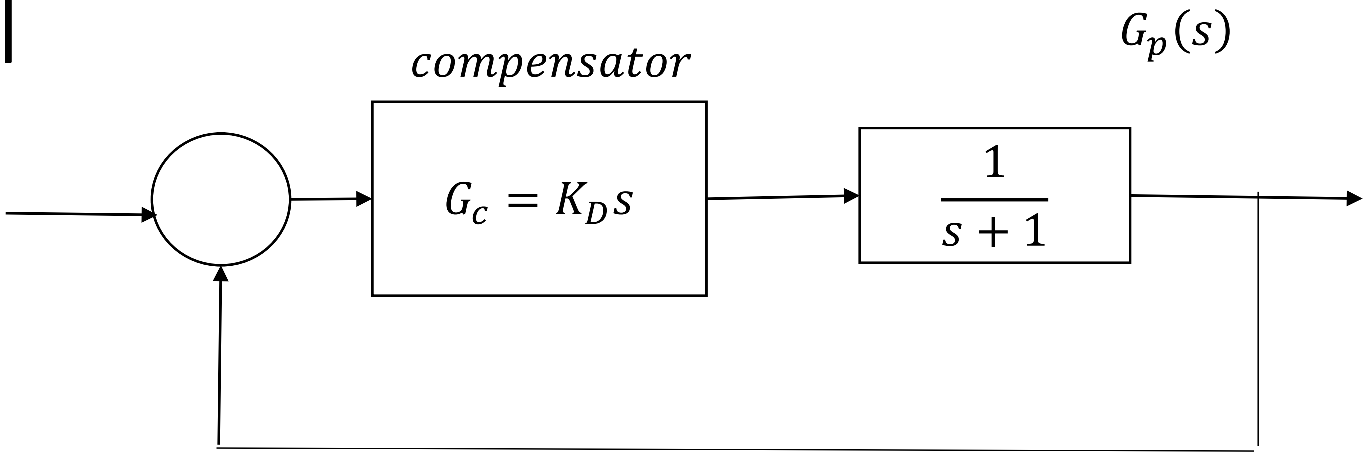


- Closed loop transfer function becomes:

$$T(s) = \frac{K_I + sKp}{s^2 + (1 + Kp)s + K_I}$$

- Combination of proportional control and integral control
  - Proportional control has impact on speed of response
  - Integral control is used to force steady state error to zero

Derivative Control



Consider derivative control
Of the first order system.

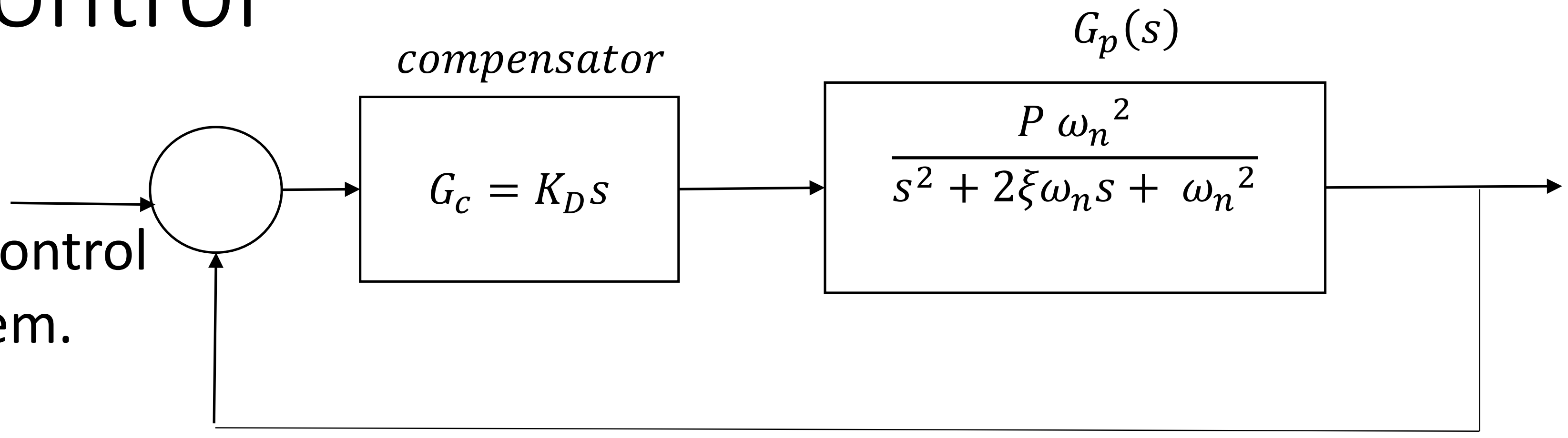
- Closed loop transfer function becomes:

$$T(s) = \frac{G_c G_p}{1 + G_c G_p} = \frac{K_D s}{(1 + K_D)s + 1}$$

- Derivative action introduces an open loop zero, as K_D increases the system may not be stable.

Derivative Control

Consider derivative control of second order system.



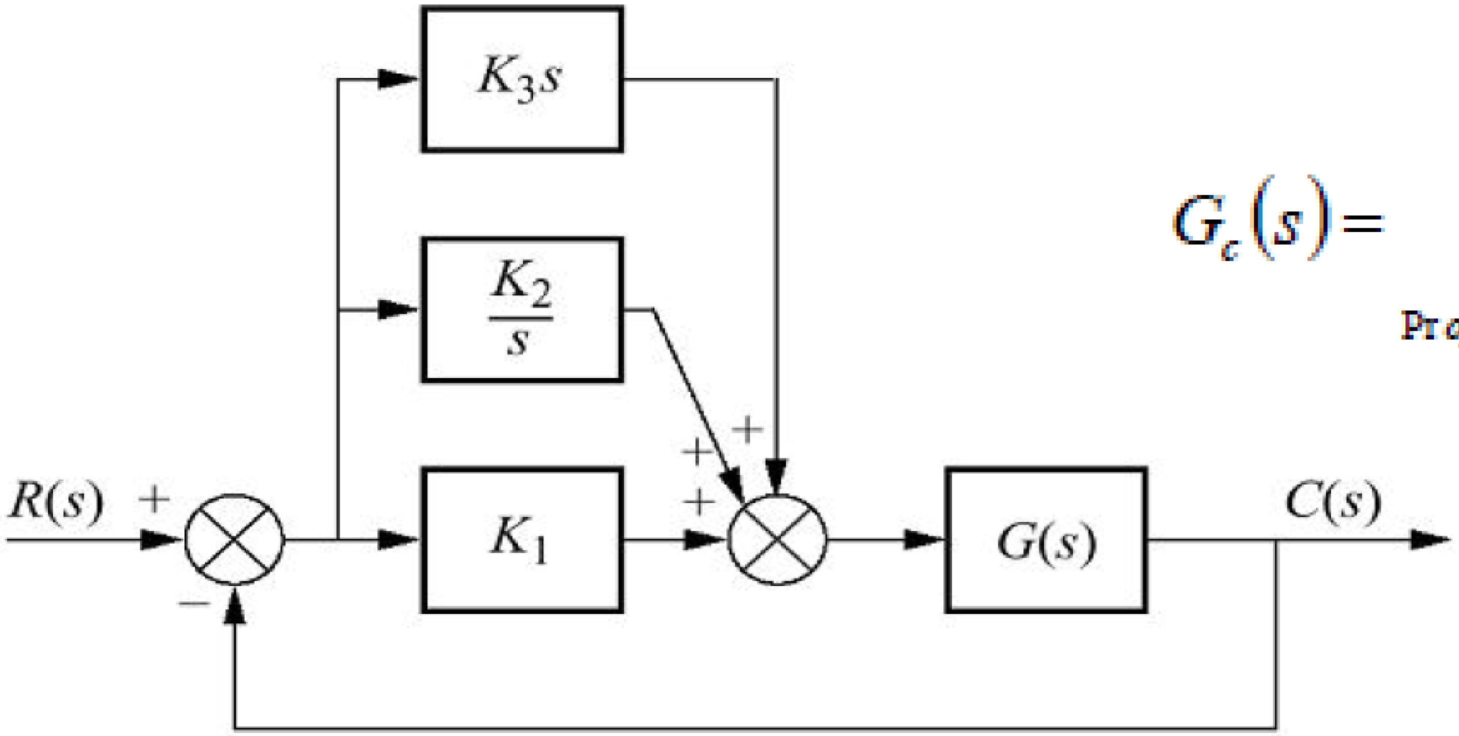
- Closed loop transfer function becomes:

$$T(s) = \frac{G_c G_p}{1 + G_c G_p} = \frac{s\ K_D\ P\ {\omega_n}^2}{s^2 + 2\left(\xi + \frac{K_D}{2} P\omega_n\right)\ \omega_n s + {\omega_n}^2}$$

- Derivative action introduces or changes damping effect, $\xi' = \xi + \frac{K_D}{2} P\omega_n$.

PID Control

- PID is an abbreviation for Proportional/Integral/Derivative Control.

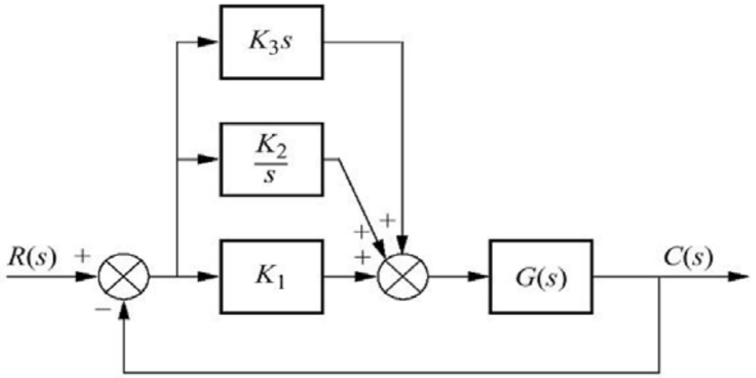


$$G_c(s) = \underbrace{K_p}_{\text{Proportional}} + \underbrace{\frac{K_I}{s}}_{\text{Integral}} + \underbrace{K_D s}_{\text{Derivative}}$$

Effects of PID Control

- Proportional Gain: steady state system error is reduced by increasing Kp.
- Integral Gain: we can use integral action to reduce steady state error to zero. The trade-off here is again stability and dynamic performance.
- Derivative Gain: Settling/peak time and damping in our system is directly affected by our derivative gain.

PID Controller



- In time domain:

$$u(t) = K_P e(t) + K_I \int_0^t e(\eta) d\eta + K_D \frac{de(t)}{dt}$$

- For proportional part, $u(k) = K_P e(k)$
- For integral part, $\frac{du(t)}{dt} = K_I\, e(t)$ ➔ $\frac{u(k)-u(k-1)}{T} = K_I\, e(t)$

$$u(k) = u(k-1) + K_I T e(k) = u(k-2) + K_I T(e(k-1) + e(k))$$

- As initial value $u(0) = 0$, the contribution from integral component can be written as:

$$u(k) = K_I T \sum_{i=1}^{k} e(i)$$

PID Controller (continued)

- The derivative part approximates to

$$u(k) = \frac{K_D}{T}[e(k) - e(k-1)]$$

- Therefore, the PID controller approximates to:

$$u(k) = K_P e(k) + K_I T \sum_{i=1}^{k} e(i) + \frac{K_D}{T}[e(k) - e(k-1)]$$

This equation is used to implement proportional, integral or derivative controllers in software!

Question

- Show that PI controller cannot stabilize $G_p(s) = \frac{1}{(s-1)^2}$