Review for Final Exam

Concepts

- Classification of real-time systems
- Differences between general propose OS and RTOS
- User space v.s. Kernel space (how to access kernel space from user space)
- Kernel module (advantages, structure to develop modules)
- Processes and Threads (pros and cons, how the address space is allocated, which segment of memory space is shared for multi-process and multi-thread programming? Be able to read C code)
- Race Condition (when it occurs? Why it is bad?)
- Poll v.s. Interrupt (pros and cons)

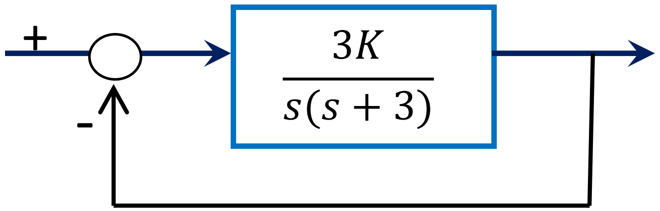
Concepts

- Scheduling Policies in Linux
- Parameters and of real time tasks (P, d, D, phase, waiting time, execution time, response time, etc)
- Representation of periodic tasks
- Cyclic Executive Scheduler (why we introduced the frame concept, what is the length of table driven CE scheduler, hyperperiod, frame size, scheduler design)
- RM, DM (How to test schedulability, conditions to apply Test1, 2, 3, sufficient and necessary condition)
- EDF (how to determine the priority of a task, scheduability test, why EDF is an optimal scheduler)
- Priority Inversion
- NPCS, PIP, PCP

Concepts

- Control Systems
  - Laplace and Inverse Laplace Transform
  - Time response and parameter of the system
  - PID Controller and software implementation
  - Z-Transform and Inverse z-transform (power series and partial fraction)
  - Z-Transfer function of sampled data, and open-loop transfer function when ZoH is included
  - Stability range in z-plane (zeros, poles, root locus, and how parameters affect the stability?)
  - Approximation of continuous control system to discrete system (forward, backward, trapezoidal Rules and Mapping between z- and s-plane)
  - Given the z-transform of a system, how to implement the discrete controller
  - Final Value Theorem

Q1: System given as

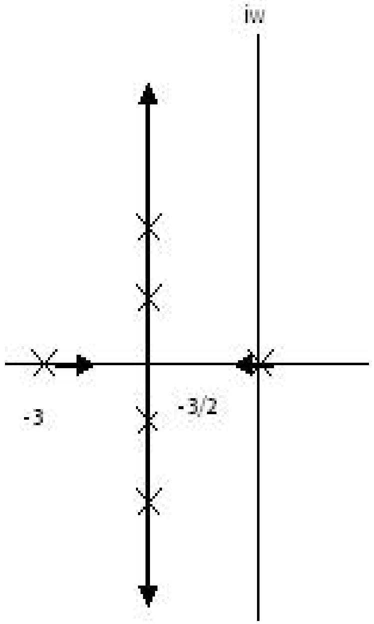


- Sketch the root locus for the closed loop feedback system. Must show the starting, branching-off and ending points of the root locus. (Consider only positive values for K)

Answer: First, find the closed-loop transfer function, $\boldsymbol{T(s)} = \frac{\boldsymbol{3K}}{\boldsymbol{s^2+3s+3K}}$. Hence, the characteristic equation is

$$s^2 + 3s + 3K \ = 0$$

The root of the characteristic equation is $s = \frac{-3 \pm \sqrt{9-12K}}{2}$, the intersection of the root locus and real axis is $9 - 12K = 0$, so $\boldsymbol{K = 0.75}$ makes $\boldsymbol{s = -1.5}$ is the break away point.



Q1 Continued

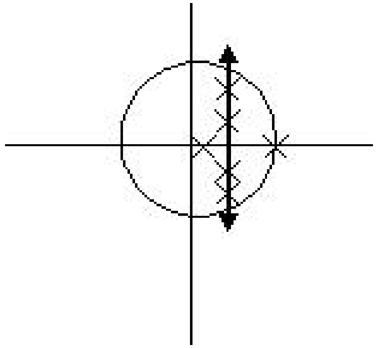
- (b) For what values of K, (if any) the system is not stable?

Answer: The system is stable for all values of K>0

- (c) An approximate closed loop z-domain transfer function of the above system was digitized assuming T = 0.3, as:

$$T(z) = \frac{3K}{11z^2 - 12z + 1 + 3K}$$

Sketch a z-domain root locus for the given discrete-time system.



Answer: The closed loop posed are given as $z = \frac{12 \pm \sqrt{144 - 44(3K+1)}}{22}$, so the break away point is $z = \frac{12}{22} = 0.55$, and Starting points are ($\frac{2}{22}$,0) and (1,0)

- (d) For what values of K, the system is stable?
- Answer: We have $z = \frac{12 \pm \sqrt{100 - 132K}}{22}$. If z has two conjugate roots, then $z = \frac{12 \pm j\sqrt{132K - 100}}{22}$. On the unit circle, $(\frac{12}{22})^2 + \frac{132K - 100}{22^2} = 1$ ➔ $K = \frac{440}{132} = 3.333$. The stable range is (0, 3.333).

Q2: Z-Transform and Inverse Z-Transform

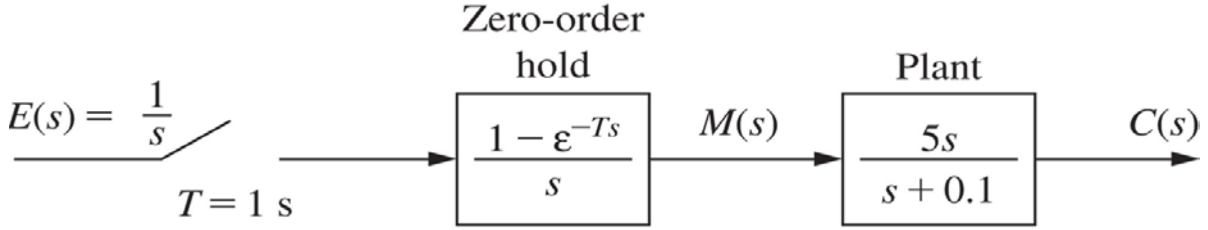
Find the z-transform of the following function $E(s) = \frac{15}{(s+2)(s+5)}$. If the sampling time T=0.1s, please find the sampled time sequence $e(nT)$ using power series method for $1 \le n \le 3$.

- Answer: $E(s) = \frac{15}{(s+2)(s+5)} = \frac{5}{s+2} - \frac{5}{s+5}$

$$E(z) = \frac{5z}{z - e^{-2T}} - \frac{5z}{z - e^{-5T}}\bigg|_{T=0.1} = \frac{1.061z}{(z - 0.8187)(z - 0.6065)}$$

$$e(T) = 1.061,$$
$$e(2T) = 1.512,$$
$$e(3T) = 1.628$$

Q3



- Find the transfer function of $C(z)$.

Answer: $G(z) = \frac{z-1}{z} Z\left[\frac{5s}{s(s+0.1)}\right] = \frac{z-1}{z} \times \frac{5z}{z-e^{-0.1}} = \frac{5(z-1)}{z-0.905}$

$$C(z) = \frac{z}{z-1} G(z) = \frac{z}{z-1} \times \frac{5(z-1)}{z-0.905} = \frac{5z}{z-0.905}$$

- Find the system response at the sampling instances $c(nT)$.

Answer: $c(nT) = 5 \times 0.905^{n\mathrm{T}}$

- Determine the input of the plant $M(s)$, then calculate $c(t)$.

Answer:

$M(s) = \frac{1}{s}$ ➔ $C(s) = \frac{1}{s}\frac{5s}{s+0.1} = \frac{5}{s+0.1}$ ➔ $c(t) = 5 \times e^{-0.1t}$ ➔ $c(nT) = 5 \times 0.905^{nT}$

Q4:

- Given the following frequency domain function: $F(s) = \frac{2s+a}{s(s-a)}$

Justify whether the Final Value Theorem can or cannot be used to find the steady state value of $f(t)$.

Answer: Not applicable, since the steady state tends to infinity.

$$\lim_{t\to\infty} f(t) = \lim_{t\to\infty}(3e^{at} - 1) = \infty$$

Q5: Solve $x(k)$, if the difference equation is given:

$$x(k) - 3x(k-1) + 2x(k-2) = e(k), where$$

$$e(k) = \begin{cases} 1, & k = 0,1 \\ 0, & k \geq 2 \end{cases}$$

$$x(-2) = x(-1) = 0$$

$$\mathrm{E}(z) = \sum_{k=0}^{\infty} e(k) z^{-k} = 1 + z^{-1}$$

Answer:

$$\mathrm{X}(z)\left(1-3z^{-1}+2z^{-2}\right)=E(z)=1+z^{-1}$$

$$\mathrm{X}(z)=\frac{z^2}{(z-1)(z-2)}\times\frac{z+1}{z}=\frac{z(z+1)}{(z-1)(z-2)}=\left(\frac{-2z}{z-1}+\frac{3z}{z-2}\right)$$

$$x(k)=-2+3\times 2^k$$

Inverse Z-Transform by Partial Fraction Expansion

- Consider a function in z-domain $\mathrm{F(z)} = \frac{2z^2+z}{z^2-1.5z+0.5}$, What is the sampled value $f(k)$.

Solution:

$$\frac{F(z)}{z} = \frac{2z+1}{z^2-1.5z+0.5}$$
$$= \frac{2z+1}{(z-1)(z-0.5)}$$
$$= \frac{A}{z-1} + \frac{B}{z-0.5}$$
$$= \frac{6}{z-1} + \frac{-4}{z-0.5}$$

$$F(z) = 6\frac{z}{z-1} - 4\frac{z}{z-0.5}$$

$$f[k] = 6u[k] - 4 \cdot 0.5^k$$

$$f = \{2, 4, 5, 5.5, \cdots\}$$

Revisit A Question in Quiz 8

Use Z transform to solve y(k), if y(k+2)-5y(k+1)+6(k)=0, where y(0)=0 and y(1)=2.

Time Shifting Property of Z-Transform

$$x(n+k) \overset{ZT}{\longleftrightarrow} z^k X(z)$$

$$x(n-k) \overset{ZT}{\longleftrightarrow} z^{-k} X(z)$$

If the initial conditions are not neglected, then

$$Z[x(n+k)] = z^k X(z) - z^k \sum_{i=0}^{k-1} x(i) z^{-i}$$

$$Z[x(n-k)] = z^{-k} X(z) + z^{-k} \sum_{i=0}^{k-1} x(-i) z^{i}$$

Solve $y[k+2] - 5y[k+1] + 6y[k] = 0$, where $y[0] = 0$, $y[1] = 2$.

$$\mathcal{Z}\{y[k+2]\} - 5\mathcal{Z}\{y[k+1]\} + 6\mathcal{Z}\{y[k]\} = 0$$

Taking z transforms:

$$z^2Y(z) - zy[0] - zy[1] - 5zY(z) + 5zy[0] + 6Y(z) = 0$$

Rearranging and using initial conditions:

$$(z^2 - 5z + 6)Y(z) = 2z$$

$$Y(z) = \frac{2z}{z^2 - 5z + 6}$$

Using partial fractions:

$$Y(z) = \frac{2}{z-3} + \frac{2}{z-2}$$

Using inverse transforms straight from the table to get the solution:

$$y[k] = 2 \times 3^k - 2 \times 2^k$$