Quiz 7

December 16, 2024 8:37 AM

① $X(k) = \left(\frac{1}{2}\right)^k [u(k) - u(k-10)]$

By def.,

$$\mathcal{Z}(X(k)) = \sum_{k=0}^{\infty} \left(\tfrac{1}{2}\right) [\underbrace{u(k)}_{\text{only }=1\text{ when }k\geq 0} - \underbrace{u(k-10)}_{\text{only }=1\text{ when }k\geq 10}] z^{-k}$$

so, this portion here is only 1 when $0 \leq k \leq 9$. Zero otherwise.

$$\boxed{\text{Geometric Series} \quad \sum_{k=0}^{n-1} ar^k = \frac{a(r^n - 1)}{r-1}}$$

$$= \sum_{k=0}^{9} \left(\tfrac{1}{2}\right)^k z^{-k} = \sum_{k=0}^{9} \left(\tfrac{1}{2z}\right)^k = \frac{\left(\frac{1}{2z}\right)^{10} - 1}{\left(\frac{1}{2z}\right) - 1} \cdot \frac{-2z}{-2z} = \boxed{\frac{2z - \left(\frac{1}{2z}\right)^9}{2z-1}}$$

② $F(z) = \dfrac{3 + 2z^{-1}}{2 + 3z^{-1} + z^{-2}} \cdot \dfrac{z^2}{z^2}$

$$= \frac{3z^2 + 2z}{2z^2 + 3z + 1}$$

$$\frac{F(z)}{z} = \frac{3z+2}{2z^2+3z+1}$$

Partial Frac Decomp.

$2z^2 + 3z + 1 = (2z+1)(z+1)$

$$\frac{A}{2z+1} + \frac{B}{z+1}$$

$$= \frac{Az + A + B2z + B}{2z^2+3z+1}$$

$A + B = 2$, $2B + A = 3$

$A = 2 - B \longrightarrow 2B + 2 - B = 3$

$\underline{A = 1} \longleftarrow \underline{B = 1}$

$$\frac{F(z)}{z} = \frac{1}{2z+1} + \frac{1}{z+1}$$

$$F(z) = \frac{z}{2z+1} + \frac{z}{z+1}$$

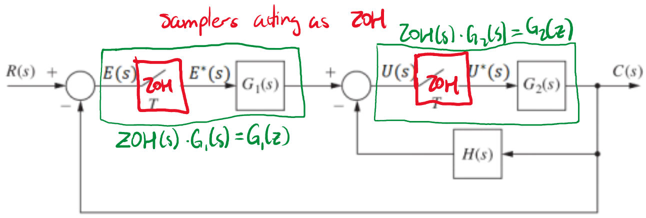
$$= \frac{1}{2} \cdot \frac{z}{z + \frac{1}{2}} + \frac{z}{z+1}$$

$$= \frac{1}{2} \cdot \frac{z}{z - \left(-\frac{1}{2}\right)} + \frac{z}{z - (-1)}$$

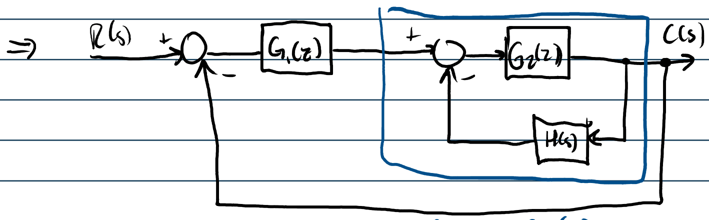
$$\boxed{f(k) = \frac{1}{2}\left(-\frac{1}{2}\right)^k + (-1)^k}$$

③ Q3: what is the system characteristic equation?

*Note: I use this notation equivalently: $ZOH(s) \cdot G(s) \Leftrightarrow G(z) \Leftrightarrow \mathcal{Z}(G(s))$



$\Rightarrow$

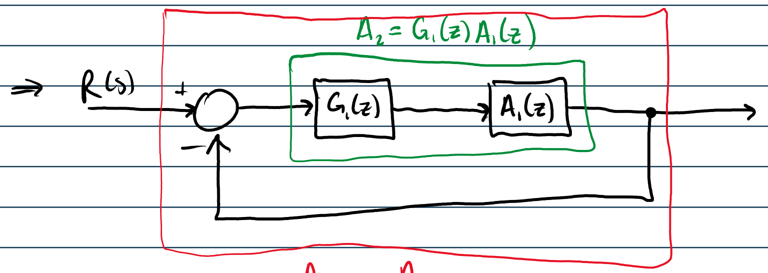


$$A_1 = \frac{G_2(z)}{1 + G_2(z)H(s)}$$

$$= \frac{G_2(z)}{1 + ZOH(s)G_2(s)H(s)}$$

$$= \frac{G_2(z)}{1+ ZOH(s)G_2(s)H(s)}$$

$$= \frac{G_2(z)}{1+\mathcal{Z}(G_2(s)H(s))}$$



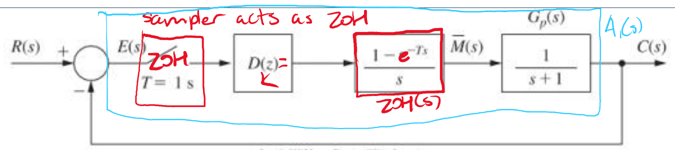
$$A_3 = \frac{A_2}{1+A_2}$$

$$A_3(z) = F(z) = \frac{G_1(z)A_1(z)}{1+G_1(z)A_1(z)} = \frac{G_1(z)\,\frac{G_2(z)}{1+\mathcal{Z}(G_2(s)H(s))}}{1+G_1(z)\,\frac{G_2(z)}{1+\mathcal{Z}(G_2(s)H(s))}} \cdot \frac{1+\mathcal{Z}(G_2(s)H(s))}{1+\mathcal{Z}(G_2(s)H(s))}$$

$$\boxed{F(z) = \frac{G_1(z)G_2(z)}{1+\mathcal{Z}(G_2(s)H(s)) + G_1(z)G_2(z)}}$$

$\therefore$, characteristic equation is $1+\mathcal{Z}(G(s)H(s)) + G_1(z)G_2(z) = 0$

④ Q4



Consider a system with T=1. The digital controller has a variable gain K.

(a) Write the closed-loop characteristic equation of the discrete system.

(b) Determine the closed-loop poles for which the system is marginally stable, and determine the corresponding K values.

(c) Consider an analog system with all sampling removed and with $G_p = \frac{K}{s+1}$. Find the range for K for which the analog system is stable.

(d) Compare the ranges of K from (b) and (c), tell the effects on stability of adding sampling component to analog system

Note 1

$$ZOH(s) = \frac{1-e^{-Ts}}{s}$$

Note 2

$$ZOH(s)\cdot G(s) = (1-z^{-1})\,\mathcal{Z}\left(\frac{G(s)}{s}\right)$$

Note 3

$$ZOH(s)\cdot G(s) = \mathcal{Z}(G(s))$$

a) $A_1(s) = ZOH(s)\cdot D(z)\cdot \underbrace{\frac{1-e^{-Ts}}{s}}_{ZOH(s)} \cdot \frac{1}{s+1}$

$= ZOH(s)\cdot D(z)\cdot ZOH(s)\cdot \frac{1}{s+1}$

$= D(z)\cdot ZOH(s)\cdot ZOH(s)\cdot \frac{1}{s+1}$

$= D(z)\cdot \mathcal{Z}\left((1-z^{-1})\,\mathcal{Z}\left(\boxed{\frac{1}{s(s+1)}}\right)\right)$

in hindsight

partial frac.

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$= \frac{As+A+Bs}{s(s+1)}$$

$$= D(z)\cdot \mathcal{Z}\left((1-z^{-1})\,\mathcal{Z}\left(\boxed{\frac{1}{s(s+1)}}\right)\right)$$

$$= \frac{As+A+Bs}{s(s+1)}$$

$A=1$

$A+B=0$

$B=-1$

in hindsight, i could've taken Z-trans. directly here; would have been easier. oops.

$$= D(z)\cdot \mathcal{Z}\left((1-z^{-1})\cdot \mathcal{Z}\left(\frac{1}{s}-\frac{1}{s+1}\right)\right)$$

$$= D(z)\cdot \mathcal{Z}\left((1-z^{-1})\left(\frac{z}{z-1}-\frac{z}{z-e^{-T}}\right)\right)$$

$$= D(z)\cdot\left(\frac{z}{z-1}-\frac{z}{z-e^{-T}}-\frac{1}{z-1}+\frac{1}{z-e^{-T}}\right)$$

$$= D(z)\cdot\left(\frac{\cancel{z-1}}{\cancel{z-1}}+\frac{1-z}{z-e^{-T}}\right)$$

$$= D(z)\cdot\left(1+\frac{1-z}{z-e^{-T}}\right)$$

$$= D(z)\cdot\left(\frac{1-z+z-e^{-T}}{z-e^{-T}}\right)$$

$$= \cancel{D(z)}^{K}\cdot\left(\frac{1-e^{-T}}{z-e^{-T}}\right) \leftarrow T=1$$

$$= K\left(\frac{0.6321}{z-0.3679}\right)$$

$$F(s) = \frac{1}{1+K\left(\frac{0.6321}{z-0.3679}\right)}\cdot\frac{z-0.3679}{z-0.3679}$$

$$= \frac{z-0.3679}{z-0.3679+0.6321K} \rightarrow \boxed{\text{Char. Eq'n} \quad 0 = z-0.3679+0.6321K}$$

b) marginally stable $\rightarrow |z|=1$

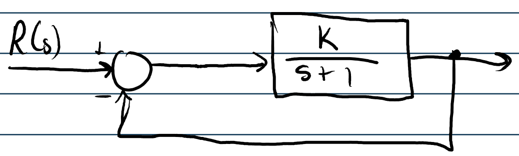
$$0 = z - 0.3679 + 0.6321K$$

$$\cancel{z}^{\pm 1} = 0.3679 - 0.6321K$$

$$0.6321 = -0.6321K \qquad -1.3679 = -0.6321K$$

$$K=-1 \qquad \boxed{K=2.164} \leftarrow \text{the value considered.}$$

c)



$$\Rightarrow \frac{1}{1+\frac{K}{s+1}}\cdot\frac{s+1}{s+1}$$

$$= \frac{s+1}{s+1+K} \rightarrow \text{char eq'n } s+1+K$$

stability $\rightarrow s \leq 0$

$$s+1+K=0$$

$$s=-1-K\leq 0$$

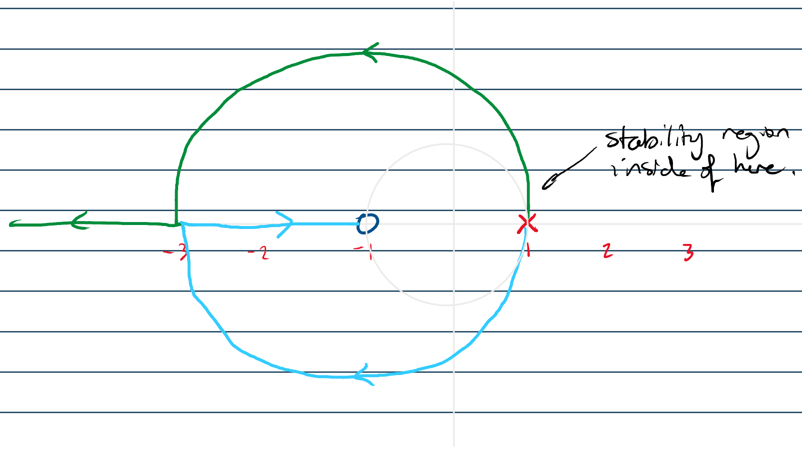
$$-K\leq 1$$

$$\boxed{K\geq -1}$$

$$\boxed{K \geq -1}$$

d) Analog system stable for all $K>0$. Digital system only stable $0<K<2.16$

⑤ $$G(z) = K\,\frac{0.05(z+1)}{(z-1)^2}$$



Breakaway pts.

$$\frac{d\,G(z)}{dz} = 0$$

$$\underline{0.05(z-1)^2 - 2(z-1)(0.05)(z+1) = 0}$$

$(z-1)^4$ don't care.

$$(z-1)^2 - 2(z-1)(z+1) = 0$$
$$z^2 - 2z + 1 - 2z^2 + 2 = 0$$
$$-z^2 - 2z + 3 = 0$$
$$z^2 + 2z - 3 = 0$$
$$(z+3)(z-1) = 0$$
$$\boxed{z = 1, -3}$$ ← breakaways.

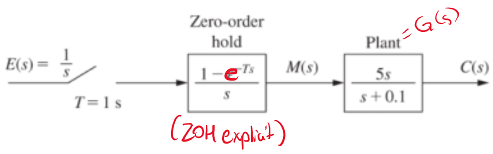
← 3 2 0

branches on real axis only to the left of here.

⑥

Q6

- For the given system:



(1) Find the output transfer function of $C(z)$.

(2) Find the system response at the sampling instances $c(nT)$.

(3) Determine the input of the plant $M(s)$, then calculate $c(t)$.

1. $$C(z) = \mathcal{Z}\left(\frac{1}{s}\right)\cdot \mathcal{Z}\left(\mathrm{ZOH}(s)\cdot G(s)\right)$$

$$= \frac{z}{z-1}\cdot(1-z^{-1})\,\mathcal{Z}\left(\frac{5\cancel{s}}{\cancel{s}(s+0.1)}\right)$$

$$= \frac{z}{z-1}\cdot(1-z^{-1})\cdot\frac{5z}{z-e^{-0.1T}}\quad (T=1)$$

$$= \frac{z-1}{\cancel{z}-1}\cdot\frac{5z}{z-e^{-0.1}}$$

$$\boxed{C(z) = \frac{5z}{z-e^{-0.1}} = \frac{5z}{z-0.905}}$$

partial frac

$$\frac{A}{s} + \frac{B}{s+0.1} = \frac{As + 0.1A + Bs}{s(s+1)}$$

$A = 0$ $\quad A + B = 5$

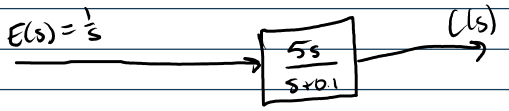
$B = 5$

oh wait i'm dumb.

2. $$\mathcal{Z}^{-1}(C(z)) = c(nT) = 5e^{-0.1nT} = 5e^{-0.1n}$$

3. I assume we assume it's an analog system for this... or it doesn't

3. I assume we assume it's an analog system for this... or it doesn't make sense. IDK anymore man.



$$C(s) = \frac{5\cancel{s}}{\cancel{s}(s+0.1)} = \frac{5}{s+0.1}$$

$$\mathcal{L}^{-1}(C(s)) = c(t) = 5e^{-0.1t}$$