COMPSCI 4ML3
Instructor: Hassan Ashtiani
Final Exam
April 2023

Surname: ____________________

Given Name: ____________________

Student Number ____________________

- This examination paper includes 9 pages (including this cover page and the last blank page) and 5 questions. **You are responsible for ensuring that your copy of the papers is complete. Bring any discrepancy to the attention of your invigilator.**
- Examination duration is 120 minutes.

Exam Instructions:

- SINGLE VERSION exam

Materials Permitted in the Exam Venue:

- No electronic aids are permitted, e.g. laptops, phones
- McMaster Standard Calculator (Casio FX-991MS/MS+)

Materials to be supplied to the students:

- Scrap paper

Instructions to the students:

- If you think there is an issue with one of the questions, make the best sensible assumption and write your assumption down along with your solution.

Grade Table

Question	Points	Score
1	45	
2	15	
3	15	
4	10	
5	15	
Total:	100	

1. For each multiple choice question, there is only one correct answer. Mark/check your choice clearly.

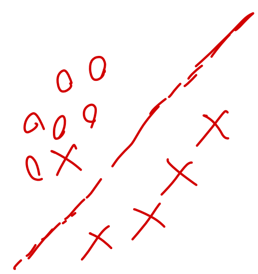
   (a) (5 points) We have a labeled data set $(x^i, y^i)_{i=1}^m$ of m points where $y^i \in \{-1, +1\}$. The following is the optimization formulation is for which model?

   $$\min_W \left(\lambda \|W\|_2^2 + \frac{1}{m} \sum_{i=1}^{m} \max\{0, 1 - y^i W^T x^i\} \right)$$

   - [ ] Regularized Least Squares
   - [ ] Hard Margin SVM
   - [x] Soft Margin SVM
   - [ ] Perceptron

   (b) (5 points) Which one is the most reasonable choice for linear classification when data is not linearly separable?

   - [ ] Using linear programming
   - [ ] Hard Margin SVM
   - [x] Soft Margin SVM
   - [ ] Perceptron

   (c) (5 points) Let TP, TN, FP, FN be the number of true positives, true negatives, false positives, and false negatives respectively. The "precision" of a model is defined by

   - [ ] $\frac{TP}{TP+TN+FP+FN}$
   - [x] $\frac{TP}{TP+FP}$
   - [ ] $\frac{TP}{TP+TN}$
   - [ ] $\frac{TP+TN}{TP+TN+FP+FN}$
   - [ ] $\frac{TP+FP}{TP+TN+FP+FN}$

   $\frac{TP}{TP+FP}$

   (d) (5 points) Which one of these choices is the most accurate about multiclass SVM?

   - [ ] The number of linear classifiers trained for one-versus-all is more than the number of linear classifiers trained for the all pairs strategy
   - [ ] There is an end-to-end version of all-pairs SVM that can be trained in end-to-end fashion
   - [x] There is an end-to-end version of one-versus-all SVM that can be trained in an end-to-end fashion

(e) (5 points) Which one is more accurate?

- ○ Universal approximation theorem indicates that you need RELU activation function rather than sigmoid to approximate any function with neural networks.
- ☒ Any bounded continuous function can be approximated (with any desired accuracy) with a neural network with one layer (i.e., one input layer and no hidden layers).
- ○ Any sigmoid neural network with 10 layers and any number of parameters can be approximated by a sigmoid network with only 1 hidden layer.

(f) (5 points) Which is more accurate regarding gradient descent (GD) and stochastic gradient descent (SGD)?

- ○ Unlike GD, SGD does not require backpropagation since the gradient is computed as the sum of gradients of individual data points.
- ○ SGD is more likely to get stuck in a local minimum since each update only uses a few data points.
- ☒ SGD makes more updates in an epoch (compared to GD)

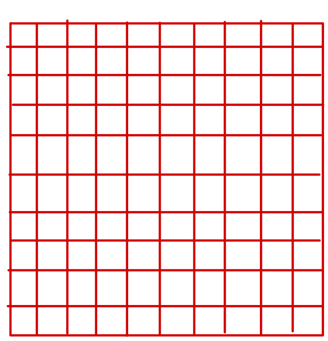
(g) (5 points) Which one is NOT a usual strategy for improving the "vanishing gradient" issue?

- ☒ Using backpropagation for computing the gradient
- ○ Using RELU activations rather than sigmoid
- ○ Using a residual architecture (ResNET)

(h) (5 points) Which one is NOT a usual strategy for regularization in neural networks?

- ○ Early stopping
- ○ Dropout
- ○ Weight sharing
- ☒ Using a ResNET architecture

(i) (5 points) We apply a 3x3 convolution kernel to a 10x10 image with no padding, and stride=2. What would be the size of the output image?



4x4

2. (15 points) Clearly write TRUE or FALSE besides each choice.

F ○ Boosting is likely to reduce overfitting.

T ○ Logistic regression is often used for classification rather than regression.

F ○ Random Forests combine the idea of boosting and bagging

F ○ Vapnik-Chrvonenkis dimension (VC dimension) of the class of all linear classifiers in $\mathbb{R}^d$ is infinite.

T ○ We have trained a fully connected neural network for classifying images and observe that its accuracy on test data is 50%. We would like to improve this to 70%. We also observe that the accuracy on training data is 85%. A sensible solution is to use dropout.

3. In this questions we want to study the relationship between smoking and having a heart attack.

   Assume that the ratio of people who smoke to the whole population is C (in other words, $\Pr[x \text{ is a smoker}] = C$ where x is a random person generated from the population).

   Assume that the probability of heart attack for smokers during their lifetime is 0.3, but this probability is 0.1 for non-smokers.

   (a) (5 points) What is the probability of having a heart-attack for a randomly selected individual (uniform over the whole population, including smokers and nonsmokers) assuming $C = 0.3$? Show your work and simplify the final solution.

$\Pr[\text{heart attack} \mid \text{smoker}] = 0.3$

$\Pr[\text{heart attack} \mid \text{non-smoker}] = 0.1$

$\Pr[\text{heart attack}] = ?$

$= \Pr[\text{heart attack} \mid \text{smoker}] \Pr[\text{smoker}]$

$+ \Pr[\text{heart attack} \mid \text{non-smoker}] \Pr[\text{non-smoker}]$

$= 0.3 \times C + 0.1 \times (1-C)$

$= 0.3 \times 0.3 + 0.1 \times 0.7 = 0.09 + 0.07 = 0.16$

(b) (10 points) We observe that from a randomly selected set of 100 people, 20 of them have experienced heart attack during their lifetime. What is the maximum likelihood estimate for the value of C in this case? Show your work and simplify the final solution.

$$\Pr[\text{heart attack} \mid c] = 0.2c + 0.1$$

$$\underset{c}{\operatorname{argmax}} \Pr[\text{dataset} \mid c]$$

$$= \prod_{i=1}^{100} \Pr[X_i \mid c] = (0.2c + 0.1)^{20} \times (1 - 0.2c - 0.1)^{80}$$

$$= \underset{c}{\operatorname{argmin}} -\log\left[(0.2c + 0.1)^{20} \times (1 - 0.2c - 0.1)^{80}\right]$$

$$= \underset{c}{\operatorname{argmin}} -20 \log(0.2c + 0.1) - 80 \log(0.9 - 0.2c)$$

$$\frac{\partial}{\partial c} = \frac{-20 \times 0.2}{0.2c + 0.1} + \frac{80 \times 0.2}{0.9 - 0.2c} = 0$$

$$= \frac{-40}{2c + 1} + \frac{160}{9 - 2c} = 0$$

$$\Rightarrow \frac{160}{9 - 2c} = \frac{40}{2c+1} \Rightarrow \frac{4}{9-2c} = \frac{1}{2c+1}$$

$$8c + 4 = 9 - 2c \Rightarrow 10c = 5$$

$$\boxed{c = 0.5}$$

4. (10 points) We are trying to fit the homogeneous line $y = a.x$ to a non-linear curve, $f(x) = x^2 + 1$ where $a, x, y \in \mathbb{R}$.

   Assume that the x values are uniformly distributed on $[0, 1]$. What should we pick for the value of a in order to minimize the squared error? Show your work and simplify your final answer.

$$\underset{a}{\operatorname{argmin}} \; \underset{x \sim U[0,1]}{E}\left[(ax - x^2 - 1)^2\right] \qquad P_X(x) = \begin{cases} 1 & x \in [0,1] \\ 0 & \text{oth} \end{cases}$$

$$\underset{a}{\operatorname{argmin}} \int_{-\infty}^{+\infty} P_X(x) \; (ax - x^2 - 1)^2 \, dx$$

$$\int_0^1 (ax - x^2 - 1)^2 \, dx$$

$$\int_0^1 a^2 x^2 + x^4 + 1 - 2ax^3 - 2ax + 2x^2 \quad dx$$

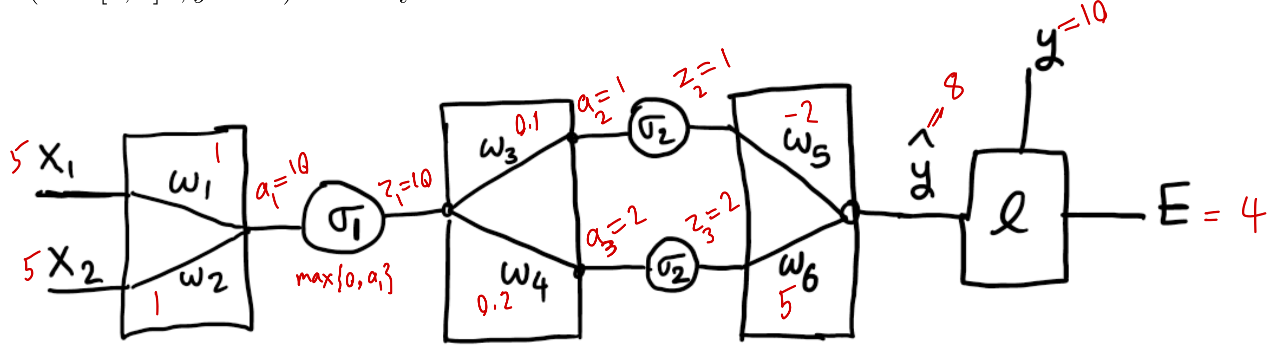
$$= \frac{10a^2 - 45a + 56}{30}$$

$$\frac{\partial}{\partial a} = \frac{20a - 45}{30} = 0 \implies a = \frac{45}{20}$$

5. (15 points) The following neural network has three layers and is used for regression. Here are the details:

   1. The activation functions of the first and second layers are ReLU (i.e., $\sigma_1(x) = \sigma_2(x) = max(0, x)$) but there is no activation function in the third layer.
   2. The loss function is the squared loss: $l(y, \hat{y}) = (y - \hat{y})^2$.
   3. $w_1 = 1, w_2 = 1, w_3 = 0.1, w_4 = 0.2, w_5 = -2, w_6 = 5$.
   4. The given input is specified by $x = [x_1, x_2]^T = [5, 5]^T, y = 10$

   Compute the partial derivative of error E with respect to w_1 for the given input point ($x = [5, 5]^T, y = 10$). Show your work.



$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{\partial (y-\hat{y})^2}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1}$$

$$= -2(y-\hat{y}) \cdot \frac{\partial \hat{y}}{\partial w_1} = -4 \frac{\partial \hat{y}}{\partial w_1}$$

$$= -4\left(\frac{\partial \hat{y}}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_1} + \frac{\partial \hat{y}}{\partial z_3} \cdot \frac{\partial z_3}{\partial w_1}\right) \qquad \hat{y} = z_2 w_5 + z_3 w_6$$

$$= -4\left(w_5 \frac{\partial z_2}{\partial w_1} + w_6 \cdot \frac{\partial z_3}{\partial w_1}\right)$$

$$= 8 \frac{\partial z_2}{\partial a_2} \cdot \frac{\partial a_2}{\partial w_1} - 20 \frac{\partial z_3}{\partial a_3} \cdot \frac{\partial a_3}{\partial w_1}$$

$$= 8 \frac{\partial a_2}{\partial w_1} - 20 \frac{\partial a_3}{\partial w_1}$$

$$= 8 \cdot \frac{\partial a_2}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1} - 20 \frac{\partial a_3}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1}$$

$$= (8 \times 0.1 - 20 \times 0.2) \frac{\partial z_1}{\partial w_1} = -3.2 \frac{\partial z_1}{\partial w_1}$$

$$= -3.2 \frac{\partial z_1}{\partial a_1} \cdot \frac{\partial a_1}{\partial w_1} = -3.2 \times 1 \times \frac{\partial a_1}{\partial w_1}$$

$a_1 = x_1 w_1 + x_2 w_2$

$$= -3.2 \times 5 = -16$$

The End