Fundamental Planetary Sciences, Updated Edition (2019)
Solutions to Selected Problems
Compiled by Jack Lissauer, with thanks to former teaching assistants Gerbs Bauer, Jenny Meyer, & Sunderrajan Krishnan, and to co-lecturers Mark Marley & Kevin Zahnle.
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Problem 1-1
(a)

Table 1: Solar System Model Parameters, 1 mm $\approx$ 1300 km

Solar System Object	Diameter	Distance	Model Object
Sun	1.1 m	–	very large beach ball
Mercury	3.7 mm	45 m	ball berring
Venus	9.5 mm	83 m	marble (slightly smaller)
Earth	10 mm	115 m	marble
Mars	5.2 mm	173 m	large ball berring
Asteroids	$\leq$ 0.8 mm	330 ± 100 m	pinch of sand and dust
Jupiter	110 mm	600 m	softball
Saturn	92 mm	1.1 km	baseball, thin paper plate (rings)
Uranus	40.1 mm	2.2 km	ping-pong ball
Neptune	38.8 mm	3.5 km	20^{th} cent. ping-pong ball
Pluto, Eris	1.7 mm	4.5, 10 km	small beads
Other TNOs	$\leq$ 1.2 mm	mostly 4 – 7 km	thimbleful of sand & dust
Proxima Cen.	160 mm	29,000 km	volleyball

One may argue that the table should include natural satellites of the planets as well, especially if Pluto and the asteroids are listed (the Galilean satellite Ganymede, for example, is slightly larger than Mercury and much larger than Pluto).

(b) The distance to Proxima Centari, 4 ly, at this scale is more than twice the diameter of the Earth! Thus, scale models are impractical.

Problem 1-4
(b) The Earth must rotate *once more than* the number of solar days since it has a prograde rotation and orbit. So the number of rotations per orbit is 366.24. The sidereal day, then is...

$$\frac{\text{Time of Orbit}}{\text{Number of Sidereal days per year}}$$
$$= \frac{(365.24\text{d} \times 24\text{h})}{366.24\text{d}} = 23.9345\text{h} = 23\text{h } 56\text{m } 4\text{s}.$$

(c) A retrograde orbit adds a *solar* day to the year. For each day in its orbit, a planet with retrograde spin needs a little *less* than a full rotation for the same part of the planet to be directly sub-solar. This is the opposite of the prograde case, where the same portion of the planet must rotate a little more following a sidereal rotation to reach the sub-solar position. Hence, a retrograde planet that rotates once per year sidereally has 2 solar days per year.

(d) The angular velocity of the Sun with respect to a fixed point on the planet is determined by the sidereal rotation velocity plus a factor involving the movement of the planet with respect to the Sun. If the planet is moving in the prograde direction, the Sun appears to move in the opposite angular direction, so that if the planet was sitting still,

$$\omega_{\odot} = -\omega_{\text{sidereal}}$$

while for prograde orbits, the apparent motion of the planet slows the motion of the Sun for a fixed point on the planet, so that

$$\omega_{\odot} = \omega_{orbit} - \omega_{\text{sidereal}}.$$

From here, using the relation $\omega = \frac{2\pi}{P}$, where P is the period of sidereal or solar rotation or orbit,

$$\frac{2\pi}{P_{\odot}} = \frac{2\pi}{P_{\text{sid}}} - \frac{2\pi}{P_{\text{orb}}}$$

and

$$P_{\odot} = \frac{1}{\frac{1}{P_{\text{sid}}} - \frac{1}{P_{\text{orb}}}} = \frac{P_{\text{sid}} P_{\text{orb}}}{P_{\text{sid}} - P_{\text{orb}}}.$$

For Mercury, $P_{\text{sid}} = 58.65\text{d}$, $P_{\text{orb}} = 87.969\text{d}$, so $P_{\odot} = 176\text{d}$.

Likewise...

Venus:

$$P_{\odot} = 116.7\text{d}$$

Note that Venus is in a retrograde rotation, and so has a negative value for the sidereal day in the formula.

Mars:

$$P_{\odot} = 24.67\text{h}$$

Jupiter:

$$P_{\odot} = 0.410080\text{d}, \; P_{\text{sid}} = 0.410042\text{d}$$

(e) To first order, the Earth, or any planet, spins at a fixed rate with respect to the background stars. The sidereal day, then, is of fixed length (for non-geological time scales, and neglecting tidal effects). The orbit of a planet, as we've seen, affects $\omega_{\odot}$, so that the longest days of the year for a prograde orbit and rotation occur when ω_{orbit} is greatest, i.e. at the Earth's perihelion (neglecting inclination and hemispherical effects).

Using Kepler's laws... (here the ω's are obital)

$$a(1-e) = \text{closest distance} \approx 0.983\text{AU}$$

$$\omega_{peri}\, r_{peri} = \bar{a}\, \bar{\omega}$$

$$\omega_{peri} = \frac{\bar{a}}{r_{peri}}\, \bar{\omega} = 1.017\, \bar{\omega}$$

$$(P_{sid} - \bar{P}_{\odot}) \cdot 1.017 = 0.0666\text{h}$$

$$0.0666\text{h} + 23.9345\text{h} = 24.0011\text{h}$$

4 seconds longer...

Note: The Earth's obliquity causes variations in the rate of apparent motion of the Sun along the equator, which also produce variations in the length of the day. The **equation of time** accounts for both types of variations and enables accurate calculation of the time using a sundial. See http://www.oarval.org/equation.htm for additional information.

Problem 2-1

(a) To go into a heliocentric orbit, the ball must be moving at escape velocity from the asteroid:

$$150 \text{ km/hr} = 0.0417 \text{ km/s} = 41.7 \text{ m/s}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}}, \ M = \frac{4}{3}\pi R^3 \cdot 3000, \ G = 6.67 \times 10^{-11} \quad \text{(in SI)}$$

$$v_{esc} = \sqrt{8000\pi R^2 G} = 41.7$$

$$R = 3.22 \times 10^4 \text{ m} \ \approx 32 \text{ km}.$$

(b)

$$\frac{1}{2}{v_o}^2 - \frac{GM}{R} = -\frac{GM}{h+R}$$

$$M = 4000\pi R^3, \ h = 50 \text{ km}$$

$$\frac{1}{2}{v_o}^2 - G4000\pi R^2 = -\frac{G4000\pi R^3}{h+R}$$

$$G4000\pi R^2 h - \frac{1}{2}{v_o}^2 R - \frac{1}{2}{v_o}^2 h = 0$$

$$R = 4.42 \times 10^4 \text{ m} \approx 44 \text{ km}.$$

(c) Periapse distance $\approx R_{asteroid}$. Periapse velocity ≈ 41.7 m/s. The lowest energy orbit is a circular orbit, so $e = 0$ and ...

$$v_c = 41.7 \text{ m/s} = \sqrt{\frac{GM}{R}}$$

$$R \approx \sqrt{\frac{41.7 \text{ m/s}^2}{4000\pi G}} = 45.5 \text{ km}.$$

Note that if the asteroid is spherically symmetric, a bound orbit is closed, and the astronaut should prepare to duck. If the asteroid is irregular, then the astronaut should stand at the point farthest from the center.

Problem 2-2

Some useful numbers:

Distances from Sun; gravity (g); mass; radius; gravitational parameter (GM)

Earth: 1 AU (1.5 $\times 10^{11}$ m); 9.81 m/s^2; 5.972$\times 10^{24}$ kg; 6378 km; 398,600 km^{3}s^{-2}

Mars : 1.52 AU (2.27 $\times 10^{11}$ m); 3.73 m/s^2; 6.421 $\times 10^{23}$ kg; 3395 km ; 42,828 km^{3}s^{-2}

The escape velocity is given by:

$v_e = (2GM/r)^{1/2}$ where G is the gravitational constant, M the mass of the planet being escaped from, r is the distance between the center of the planet and the point at which escape velocity is being calculated, here the top planet's atmosphere ($r = R_p$ = radius of the planet)

Mars: $v_e = 5.022$ km/s

Earth $v_e = 11.18$ km/s

(a) Mars to Earth

The orbital velocity of Mars (circular orbit approximation) is $v_c = (GM_\odot/1.52\text{AU})^{1/2} =$ 24.16 km/s

To go from the orbit of Mars to Earth, the semimajor axis must be the average of those of Mars and Earth: (1+1.52)AU/2 = 1.26 AU (1.496×10^8 + 2.274×10^8 km) / 2 = 1.888×10^8 km

The specific (per unit mass) energy must, therefore, be 1.52/1.26 = 1.2063 $\approx$ 1.2 times as large as that of a circular orbit at Mars's orbit. The specific potential energy is the same as that of a circular orbit at Mars's orbit, $-v_c^2$. The specific kinetic energy of the particle is $0.5v_p^2$, so $0.5v_p^2 - v_c^2 = -0.6v_c^2$

$v_p = (0.8v_c^2)^{1/2}$

The velocity relative to Mars is thus $v_\infty = v_c - v_p = 2.55$ km/s

Therefore to escape Mars with this velocity, it must leave the martian atmosphere at a transfer velocity v_t given by: $v_t = (v_\infty^2 + v_e^2)^{1/2} = 5.63$ km/s

(b) The orbit is the same as for the minimum energy trajectory from Earth to Mars, so by the calculation given for part **(c)**: $v_{impact} = 11.67$ km/s

(c) Earth to Mars

The orbital velocity of Earth (circular orbit approximation) is:

$v_c = (GM_\odot/1\text{AU})^{1/2} = 29.78$ km/s.

To go from Earth's orbit to Mars, the semimajor axis must be the average of those of Earth and Mars: (1+1.52)AU/2 = 1.26 AU

The specific energy must, therefore, be 1/1.26 times that of a circular orbit, so $0.5v_p^2 - v_c^2 = -v_c^2/2.52$

$v_p = (2 - (1/1.26))^{1/2} v_c = 1.098 v_c$

The velocity relative to Earth is thus $v_\infty = v_p - v_c = 2.93$ km/s

$v_t = (v_p^2 + v_e^2)^{1/2} = 11.67$ km/s

(d) The orbit is the same as for the minimum energy trajectory from Mars to Earth, so $v_{impact} = 5.63$ km/s.

(e) Planetary rotation reduces the minimum required energy, because the atmosphere is at moving (rotating with the planet) and at optimal times its motion is in the correct direction; maximum motion is at the equator. Eccentricity also reduces the minimum required velocity, with the preferred configuration being escape when the source planet at periapse (where its orbital velocity is largest, so greatest change in orbital energy for a given velocity relative to the planet) and impact when the destination planet is closest to the semimajor axis of the outer planet (this requires optimal alignment of the periapse angles of the two planets). Inclination has negligible effect since the inclinations are small and effects are second-order in inclination.

Problem 2-14

As the Moon orbits the Earth in the prograde direction with an orbital period longer than 1 siderial day, $P_s = (P_r^{-1} - P_o^{-1})^{-1} = ((366.24/365.24) - (27.32)^{-1})^{-1} = 1.035$ days $= 24.84$ hours.

Problem 2-16

(a) The force due to tides is linearly proportional to the mass of the perturber, so the height of the tide would be reduced by a factor of 2.

(b) The height of the tide is proportional to the mass of the perturber divided by the cube of the distance. Thus, the Sun produces tides 0.41 times as large as those of the Moon. So since the Moon's tides drop by a factor of 2 and the Sun's don't change, the total tides are $0.91/1.41 = 0.65$ times as large (reduced by 35%).

Problem 3-4

$$\sum \langle E_G \rangle = -\frac{Gm_1 m_2}{r}$$

$$-2\sum_{i=1}^{2} \langle E_K \rangle = -2\sum_{i=1}^{2} \frac{1}{2} m_i {v_{circ}}^2$$

$$v_{i,circ} = \sqrt{\frac{GM}{r}\frac{m_{3-i}}{M}}$$

$$\Rightarrow -2\sum \langle E_K \rangle = \frac{1}{2} m_1 {v_1}^2 + \frac{1}{2} m_2 {v_2}^2$$
$$= -2\frac{1}{2}\frac{GM}{rM^2}[{m_2}^2 m_1 + {m_1}^2 m_2] = -2\frac{1}{2}\frac{GM^2}{rM^2} m_2 m_1$$
$$= -\frac{Gm_1 m_2}{r} = \sum \langle E_G \rangle$$

Problem 3-7

(a) The hot big bang produced many particles that decayed into protons, which are H nuclei. Most of these protons never fused into heavier elements. He has two protons, and thus was the first element formed by fusion of H after the big bang and also in stars. (Also, since no element with atomic mass 5 or 8 is stable, it is very difficult to produce heavier elements by combining ^{4}He with either other ^{4}He or with protons.)

(b) C can be formed in dense environments by fusion of 3 alpha particles (^{4}He nuclei), and O by addition of another alpha particle or 4 protons (2 of which subsequently decay into neutrons). C and O have more binding energy per nucleon, therefore are more stable, especially at high temperature, than are Li, Be & B.

(c) ^{56}Fe has more binding energy per nucleon than any other isotope, i.e., it is the most stable nucleus.

(d) Only light elements were produced in the big bang, and since iron is the most stable element, only endothermic nuclear reactions produce heavier elements.

(e) Odd atomic number generally have less binding energy per nucleon than do neighboring elements of even atomic number, i.e., even elements are more stable than odd ones.

Problem 4-6

Different parts of a real planet have different temperatures, T; therefore they would have different peaks on the emission-frequency curve. Thus, the spectrum of emitted radiation from a planet is broader than a blackbody spectrum.

Problem 4-9

(a) Use Equation (4.17) and Tables E.9 and E.10:

$$T_{eq} = \left(\frac{\mathcal{F}_\odot}{r_{AU}^2} \frac{(1 - A_b)}{4\epsilon\sigma} \right)^{1/4}$$

- Earth

$$A_b = 0.306 \text{ and } r_{AU} = 1 \Rightarrow T_\oplus = 261.2 \text{ K}$$

- Jupiter

$$A_b = 0.343 \text{ and } r_{AU} = 5.2 \Rightarrow T_J = 113 \text{ K}$$

(b) Use equation (4.8):

$$\lambda_{max} = \frac{2.89 \times 10^{-3}}{T} \text{ m}$$

- For Earth

$$T_\oplus = 255.3 \text{ K} \Rightarrow \lambda_{max} = 11.1\ \mu\text{m}$$

- For Jupiter

$$T_J = 113 \text{ K} \Rightarrow \lambda_{max} = 25.7\ \mu\text{m}$$

From Figure 4.1, it is apparent that these peaks are in the middle of the infrared. Astronomers call this region the ‘mid-infrared’.

Problem 4-14

(a) Large sheets of ice would be much brighter than oceans, land, or vegetation and that would increase the albedo.

(c) Many answers are reasonable here. Say we choose 0.8; then by equation (4.17) we get 191 K or −82 C.

(d) As the world got colder more ice would form and the albedo would keep increasing. Ignoring any other feedback effects this would be an unstable change. In the real world as ice freezes volcanic CO_2 would still be pumped in to the environment and not lost to weathering, eventually the greenhouse effect would overcome the albedo effect.

Problem 4-17

(a) Substituting into equation (4.17) gives $T_{\text{eq}} = 167.8\,\text{K}$. When the planet has no internal sources of heat, $T_{\text{eff}} = T_{\text{eq}}$.

(b) At the edge of the atmosphere, optical depth $\tau = 0$, $T_0^4 = 0.5 T_{\text{eff}}^4$. Thus $T_0 = 141.1\,\text{K}$.

(c) The temperature at a given optical depth in the atmosphere is given by

$$T^4(\tau) = T_0^4 (1 + \frac{3}{2}\tau).$$

At $\tau = 2/3$ we have $T^4 = 2T_0^4 = T_{\text{eff}}^4$. Thus the continuum emergent flux appears to emerge with a spectrum characteristic of T_{eff}.

(d) Using the above relation and setting $\tau_{\text{surf}} = 10$ we have $T_{\text{surf}}^4 = 16 T_0^4$ or $T_{\text{surf}} = 2T_0 = 282\,\text{K}$.

Problem 5-5

For gases, by the ideal gas equation, density is proportional to gas particle mass. Molecular nitrogen has a molar mass of 28, and molecular oxygen has a molar mass of 32. A pure oxygen-nitrogen mixture would have a molar mass between 28 and 32. Water vapor has a molar mass of 18, which makes it lighter and therefore less dense than any nitrogen-oxygen mixture. Thus, if we replace a portion of the $N_2 - O_2$ mixture in a parcel of air with water vapor, the parcel would rise since the density would be lower.

Problem 5-10

Ozone is formed in a reaction that begins with the photodissociation of molecular oxygen: $O_2 + h\nu \rightarrow O + O$. Ozone is then formed by 3-body reactions: $O + O_2 + M \rightarrow O_3 + M$. At very low pressures, three-body reactions are very rare and O_3 production is unlikely. At high pressures, the column of gas above absorbs most of the incident UV flux. Thus, it seems reasonable to expect that there is some altitude where both ozone production can proceed efficiently because there are sufficient collisions and there is adequate UV flux to drive the reaction.

Problem 5-18

(a) CO_2 in the atmosphere is absorbed by the oceans, and dissolved into water as carbonic acid promotes the weathering of rocks, thereby forming carbonate. The rate of both reactions are dependent on the temperature. If the temperature increases, the reaction rates increase thereby decreasing the amount of CO_2 in the atmosphere. Since CO_2 is a greenhouse gas, a decrease of CO_2 causes a decrease in the surface temperature as less heat is trapped in the atmosphere. Thus, the surface temperature is restored. Conversely a fall in temperature leads to more sluggish reactions and a gradual buildup in atmospheric CO_2 and eventual warming.

(b) Deposits of carbonate rocks on the ocean floor may be subducted in subduction zones, thus releasing the carbon dioxide to the atmosphere through island arc volcanic activity. Likewise extensive continental volcanism, such as widespread basaltic lava flows, can release carbon dioxide trapped in continental carbonates. In these cases, the releases of carbonate art not necessarily coupled to the climate.

Problem 5-19

(a) Soot absorbs sunlight, and it has a much higher optical depth to visible radiation from the Sun than to the longer-wavelength thermal IR radiation from Earth, so the radiation is absorbed above an altitude where significant greenhouse warming occurs. When sunlight is absorbed so high in the atmosphere, the energy does not reach the surface and therefore the surface cools. This resembles the surface cooling caused by increased cloud cover or simply a cloud passing in front of the Sun. This is often called the “anti-greenhouse effect.” The anti-greenhouse effect resembles the cooling caused by increasing albedo, but differs from albedo in that sunlight is absorbed and then re-radiated as infrared energy, rather than simply being reflected.

Problem 6-12

(a) This just entails tabulating the acceleration of gravity g and escape velocity for each world. Then to compute the exit velocity, just equate the kinetic energy at time $t = 0$ with the potential energy at the height of the ballistic arc when $v = 0$, so $mgy = 0.5mv^2$. Velocities range from 31 m/s for old faithful to 240 m/s for Enceladus to 450 m/s for Io. Then it is easy to express as a fraction of escape velocity.

(b) There is a big spread in ratios, so it is probably not the same mechanism that is producing each type of geyser (which we know to be the case). Also, ballistics is not the only mechanism propelling the geysers.

Problem 6-14

(a) Using eq. (6.6), and inserting numerical values with SI units: $2 \times 3000^{0.11} \times 3500^{-0.33} \times 9.8^{-0.22} \times 500^{0.12} \times (0.5 \times 3000 \times (4\pi/3)500^3 \times (1.5 \times 10^4)^2)^{0.22} \times (2^{-1/2})^{1/3} = 1.06 \times 10^4$ m = 10.6 km.

Problem 6-15

(a) A meteoroid is slowed substantially if it passes through its mass of gas. For Earth, the mass per unit area of the atmosphere is slightly more than 10^4 kg/m^3, so an iron meteoroid smaller than $\sim$ 1 meter in radius is slowed appreciably.

(b) For Venus, the mass per unit area of the atmosphere is slightly more than 10^6 kg/m^3, so an iron meteoroid smaller than $\sim$ 100 meters in radius is slowed appreciably.

(c) Minimum crater sizes can be estimated using eq. (6.6) with $\sin i = 1$ and the above estimates for impactor mass and an impact speed of 10 km/s. For Earth, a 1 meter radius iron body produces a crater $\sim$ 80 meters in diameter. For Venus, a 100 meter radius iron body produces a crater $\sim$ 3 km in diameter. Craters smaller than these radii are rare.

Problem 6-16

(e) Random statistical variations limit the precision to which craters can be used to determine relative dates, especially if the number of craters is small.

Problem 7-1

The ratio can be computed using the blackbody radiation formula (eq. 4.11): $(4500/5800)^4 = 0.36$.

Problem 7-6

Many answers are acceptable, for instance:

(a) Satellite communication systems are damaged or destroyed by greater radiation above atmosphere. Radio communication would be affected by change in the ionosphere.

(b), **(c)** More radiation reaches surface, so radiation-intolerant organisms die and mutation rate increases. Organisms that use magnetic fields to orient themselves or to navigate during migration adapt or die out.

Problem 8-4

(a) Jupiter and Saturn both emit substantially more energy than they absorb from the Sun, implying large internal heat sources. Radiation and conduction would require extremely large temperature gradients to transport enough energy from the interiors of these planets to the region at which it could be radiated to space, and such gradients would make the planets' interiors convectively unstable. Convection only requires slightly super-adiabatic temperature gradients to transport the required energy. Thus, in the regions of the planets' envelopes deeper than several optical depths to outgoing radiation, the temperature should increase with depth approximately adiabatically.

(c) Uranus does not have a substantial internal heat source, so the temperature gradient with depth could be shallower than the adiabatic rate.

Problem 9-8

(a) Neglecting the small variations of g within each atmosphere, atmospheric mass can be calculated as:

$$M = P \times 4\pi R^2/g.$$

(Surface pressure multiplied by the area gives the force, divide by g to get mass.)

For Earth, using $P = 101.3$ kPa and $R_\oplus = 6370$ km, $M_{atm} = 5 \times 10^{18}$ kg.

For Venus, $P = 9322$ kPa, $R = 6052$ km, so $M_{atm} = 5 \times 10^{20}$ kg.

(b) A 3 km layer of water has the mass:

$4\pi R^2 \rho \Delta R = 1.4 \times 10^{21}$ kg.

(c) Without considering water, the atmosphere of Venus is 100 times as massive. Adding in the water content in the Earth's oceans, Earth's atmosphere + hydrosphere is about 3 times as massive as that of Venus.

Problem 9-9

There are many possible answers. The physical lines of evidence are the small valley networks that appear to have formed by running water acting over a long period of time. (The huge outflow channels, which seem to have formed catastrophically, are not a good answer as they could form even under todays climate if there was an appropriate source of water.) The best chemical evidence for past water comes from the Mars rovers. There are many examples, but two include the presence of sulfate-rich minerals (which required long-lasting soaking in acidic water to form) and the presence of "concretions" or small marble-sized mineral spheres that also form by soaking in water.

Problem 10-4
(a) The global average flux at Titan is $1 \times 10^{16} \div 9.5^2 \div 4 = 2.77 \times 10^{13}/\mathrm{m}^2/\mathrm{s}$.

(b) H_2 escape flux $= 2.77 \times 10^{13}\mathrm{m}^2/\mathrm{s}$
C_2H_6 formation rate $= 2.77 \times 10^{13} \div 2 = 1.38 \times 10^{13}\mathrm{m}^2/\mathrm{s}$
CH_4 destruction rate $= 1.38 \times 10^{13}\mathrm{m}^2/\mathrm{s}$.

(c) The number of moles of gas in a column of Titan's atmosphere is:
$N = 1.5 \times 10^5 \div 0.028 \div 140 = 3.83 \times 10^6$ moles/m^2.

The column density (the number of molecules in a column) of methane is:
$0.05 \times 3.83 \times 10^6 \times 6 \times 10^{23} = 1.15 \times 10^{29}$ CH_4 molecules/m^2.

The lifetime of the current reservoir of CH_4 in Titan's atmosphere is roughly:
$1.15 \times 10^{29} \div 2.77 \times 10^{13} = 4.14 \times 10^{15}$ sec = 130 million years.

(d) $1.15 \times 10^{29} \div 2 \div 6 \times 10^{23} = 9.6 \times 10^4$ moles/m^2.
An alternative derivation is: $0.05 \times 3.83 \times 10^6 \div 2 = 9.6 \times 10^4$ moles/m^2.

(e) 180 meters.

(f): CH_4: (bp) 111 K; (fp) 90.7 K
C_2H_6: (bp) 184.6 K; (fp) 90.4 K.

(g) Cassini-Huygens scientists expected to land in a liquid methane-ethane "ocean" more than 200 m deep, because photochemical decomposition of methane over 4.56 yrs would have left at least 180 m of liquid ethane at the surface. Imaging in infrared wavelengths showed features that resembled rivers, lakes, oceans, etc, but they actually found land, probably because not all the radar-dark features that resembled lakes or seas are actually liquid bodies; smooth surfaces could also look similar. However, there are actually hydrocarbon liquid lakes on Titan, mostly at high latitudes.

Problem 11-1
Since the eccentricity of the Earth's orbit is very small, we consider the orbit to be circular with radius equal to 1 AU. Orbital velocity of Earth is therefore given by:

$$v_\oplus = \sqrt{\frac{\mathrm{GM}_\odot}{a_\oplus}} = 29.8 \text{ km/s}.$$

When the meteoroid approaches the Earth's atmosphere, its potential energy would be converted into kinetic energy resulting in the velocity of entry given by:

$$v_{entry} = \sqrt{v_\infty^2 + v_{esc}^2}, \tag{1}$$

where v_{esc} is the escape velocity of Earth equal to 11.2 km/s.

(a) For a meteoroid with a similar orbit as Earth, $v_\infty = 0$ giving $v_{entry} =$ 11.2 km/s.
(b) Inclination $i = 180°$ gives $v_\infty = 2v_\oplus = 59.6$ km/s giving $v_{entry} = 60.6$ km/s.

Problem 11-8

The half-life of ^{234}U (2.47×10^5 years) is much shorter than the age of the Earth (≈ 4.55 Gyr). Thus, any ^{234}U present now must be in dynamic equilibrium with ^{238}U.

$$\frac{dN_{234}(t)}{t} = \frac{N_{238}(t)}{\gamma_{238}} - \frac{N_{234}(t)}{\gamma_{234}} = 0,$$

giving,

$$\frac{N_{234(t)}}{N_{238}(t)} = \frac{\gamma_{234}}{\gamma_{238}} = \frac{2.47 \times 10^5 y}{4.51 \times 10^9 y} = 0.0055\%$$

As we ignore the small amount of ^{235}U, the percentage abundance of ^{234}U is 0.0055%.

Problem 12-3

Using eq. (2.22), the energy of the comet per unit mass is given by:

$$\frac{E}{m} = -\frac{GM_\odot}{2a} = \frac{v^2}{2} - \frac{GM_\odot}{r}.$$

Multiplying by $-2/(GM_\odot)$ and inverting gives (inserting numbers in SI units):

$$a = \left(\frac{2}{r} - \frac{v^2}{GM_\odot}\right)^{-1} = \left(\frac{2}{1.496 \times 10^{11}} - \frac{40,000^2}{6.674 \times 10^{-11} \times 1.989 \times 10^{30}}\right)^{-1} = 7.6 \times 10^{11},$$

which equals 5.08 AU.

Problem 12-4

(a)

$$N(R) = N_o \left(\frac{R}{R_o}\right)^{-\zeta}$$

$$M(R) = N(R)\frac{4\pi\rho}{3}R^3$$

pick an arbitrary value of R_1. For $\zeta = 4$, we have

$$M(\frac{R_1}{2} < R < R_1) = \int_{\frac{R_1}{2}}^{R_1} N_o \frac{4\pi\rho}{3} r^{-1} R_o dr = \frac{4\pi\rho}{3} N_o R_o^4 \ln 2$$

independently of R_1, so mass is shared equally among logarithmic intervals in radius.

(c) The condition is that $\int_{\frac{R_1}{2}}^{R_1} A(r)dr$ be independent of R_1. Omitting the constants we have

$$\zeta \neq 3:$$

$$\int_{\frac{R_1}{2}}^{R_1} r^{2-\zeta} dr = \frac{r^{3-\zeta}}{3-\zeta}\Big|_{\frac{R_1}{2}}^{R_1} = \frac{1 - \frac{1}{2}^{3-\zeta}}{3-\zeta} R_1^{3-\zeta}$$

$$\zeta = 3:$$

$$\int_{\frac{R_1}{2}}^{R_1} r^{-1}dr = \ln r\Big|_{\frac{R_1}{2}}^{R_1} = \ln 2$$

independent of R_1, so $\zeta = 3$.

Problem 12-10

Set the rotational speed at the equator equal to the speed of a circular orbit at this distance:

$$\frac{2\pi R}{P_{rot}} = \sqrt{\frac{GM}{R}} = \sqrt{\frac{4}{3}\pi G R^2 \rho}$$

So

$$P_{rot} = \sqrt{\frac{3\pi}{G\rho}} \approx 6860 \text{ s} \approx 1.9 \text{ hours.}$$

Note that the same result may be obtained by setting the centrifugal force per unit mass equal to gravity at the equation:

$$F_c = \frac{v^2}{R} = \omega^2 R = \left(\frac{2\pi}{P_{rot}}\right)^2 R,$$

$$F_g = \frac{GM}{R^2} = \frac{4}{3}\pi G R\rho,$$

so

$$\left(\frac{2\pi}{P_{rot}}\right)^2 R = \frac{4}{3}\pi G R\rho$$

etc. as before.

Problem 12-19

(a) Using the Keplerian approximation,

$$a = \frac{1}{2}(r_{ap} + r_{peri}) = 8 \text{ AU}$$

$$P_{\text{years}} = a_{\text{AU}}^{3/2} \approx 22.6 \text{ years} \approx 7.15 \times 10^8 \text{ s.}$$

(b) Assume 1.5 AU is the proper average distance to use for the time period of solar exposure, and ignore the $1/r^2$ effects of the flux variation over the orbit for now. From equation (12.3) . . .

$$Q = (3 \times 10^{17})\pi \frac{R^2}{r_{\text{AU}}^2} \text{ molecules/s}$$

$$h_{loss} = \frac{Q}{4\pi R^2 \rho} \cdot \frac{P_{orb}}{10} \cdot \frac{18}{N_o} \text{ cm} \approx 71 \text{ cm}$$

where P_{orb} is the period of the comet, ρ is its density, and N_o is Avagadro's number, 6.02×10^{23} atoms per mole.

Problem 13-2

(a) Assuming $M_p \gg M_{m_{1,2}}$, Equation (13.4) still applies ...

$$g_{eff} = GM_p\hat{\mathbf{r}}(\frac{r}{a^3} - \frac{1}{r^2}),$$

but here the center of the moon must be replaced by the center of mass of the two satellites.

$$\frac{dg_{eff}}{dr} = GM_p(\frac{1}{a^3} + \frac{2}{r^3}) \approx \frac{3GM_p}{a^3}$$

$$\frac{G(M_{m_1} + M_{m_2})}{(R_{m_1} + R_{m_2})^2} = \frac{3GM_p}{a^3}(R_{m_1} + R_{m_2}).$$

For Moons of equal sizes and mass:

$$\frac{2GM_m}{8{R_m}^3} = \frac{3GM_p}{a^3}$$

$$R_m = (\frac{M_m}{12M_p})^{1/3}a$$

$$\Longrightarrow a = \left(\frac{12\rho_p}{\rho_m}\right)^{1/3} R_p \approx 2.29 \left(\frac{\rho_p}{\rho_m}\right)^{1/3} R_p$$

which is identical to the Roche limit except for the constant.

(b) This constant is larger than in the case of a single spherical moon, because the two bodies are more stretched out, so tidal forces are larger. However, the magnitude of this effect is not as large as that of deformation of a fluid moon.

Problem 13-3

The Roche limit is given by equation (13.8):

$$a_R = 2.456(\frac{\rho_p}{\rho_m})^{1/3}R_p$$

$$a_{R,Jup} = (1.89 \times 10^5)(10{\rho_m}^{-1/3})\text{km},\ a_{R,Sat} = (1.26 \times 10^5)(10{\rho_m}^{-1/3})\text{km},$$
$$a_{R,Ura} = (6.75 \times 10^4)(10{\rho_m}^{-1/3})\text{km},\ a_{R,Nep} = (7.13 \times 10^4)(10{\rho_m}^{-1/3})\text{km}$$

where ρ_m is in units of kg/m^3.

If we think of most satellites as “dirty ice balls”, then $\rho_m \approx 1000$ and $10{\rho_m}^{-1/3} \approx 1$. Jupiter’s rings span from half the Roche limit value calculated using the ice ball approximation to nearly that value. Three minor satellites lie interior to the Roche limit, but the Galilean satellites lie outside twice this value. Saturn’s main rings extend to just beyond the Roche limit for ice-ball bodies, while the G-ring and the faint E-ring extend well beyond it. To complicate matters, there are numerous small satellites near this calculated value. For Uranus and Neptune there are several moons which reside within the ice-ball approximation Roche limit, hence the satellite densities may be greater. The rings for the outer two planets fall well within this $a_{R,ice}$ value. Hence, a single density “dirty ice ball” model does not seem to suffice.

Problem 13-7

(a) Neglecting Saturn's oblateness and applying Kepler's 3rd law gives

$$P = \sqrt{\frac{4\pi^2 a^3}{GM_{Saturn}}}$$

$$\Delta P = \sqrt{\frac{4\pi^2}{GM_{Saturn}}}({a_o}^{3/2} - {a_i}^{3/2}) \approx \sqrt{\frac{4\pi^2 a^3}{GM_{Saturn}}}(\frac{3}{2}\frac{\Delta a}{a})$$

$$n = \frac{P}{\Delta P},\ t = nP = \frac{P^2}{\Delta P}$$

i)

$$P \approx 2.373 \times 10^4 \text{ s},\ \Delta P = 0.445 \text{ s}, t \approx 1.27 \times 10^9 \text{ s} \approx 40 \text{ years}$$

ii) Δa is 100 times larger, thus ΔP is 100 times smaller so

$$t \approx 1.27 \times 10^7 \text{ s} \approx 0.4 \text{ years.}$$

Problem 14-3

(a)

$$P_{tr} = (R_\star + R_p)/a(1 - e^2) \qquad (14.5)$$

$$R_\star = 6.95 \times 10^8 \text{m}.$$

For Jupiter: $R_J = 7.15 \times 10^7$ m; $a_J = 778.57 \times 10^6$ km; e_J= 0.0484; P_J = 0.0987%.

For Venus: R_V = 6.05×10^6 m; $a_V = 108.21 \times 10^6$ km; e_V= 0.0067; P_V= 0.64%

(b) Consider an observer who lies in Jupiter's orbital plane, and thus views Jupiter to make a central transit. As it orbits the Sun, the elevation of Venus is sinusoidal in time and has an amplitude of $2.3° = 0.04$ radian, so it moves up and down by 4.344×10^6 km, which is 6.24 times the size of the Sun as viewed from Venus. The fraction of time that the magnitude of the sine function is below 1/6.24 is arcsin (.16 radians) = 0.16.

Thus, neglecting the small effects of planetary sizes, eccentricities and the small deviation the observer can be from the plane of Jupiter's orbit, $\sim 16\%$ of the observers who view Jupiter to transit will also see Venus to pass in front of the Sun.

Problem 14-8

(a) The fraction of the Sun's radiation that is reflected by Earth equals the cross-sectional area of the Earth multiplied by Earth's albedo and divided by the area of a sphere of radius equal to Earth's orbit:

$$\frac{\pi R_\oplus^2}{4\pi r_\oplus^2} \times \mathcal{A}_\oplus = \frac{0.3}{4} \times (\frac{6370}{1.5 \times 10^8})^2 \approx 1.3 \times 10^{-10}.$$

(b) The fraction of the Sun's total energy emitted by Earth in the thermal infrared is

$$\frac{4\pi R_\oplus^2 \sigma T_{\text{eff},\oplus}^4}{4\pi R_\odot^2 \sigma T_{\text{eff},\odot}^4},$$

which can be multiplied by the fraction of the Sun's luminosity that is emitted in this wavelength range.

Alternatively, take the ratio of surface areas of the bodies and multiply by the ratios of temperatures (Rayleigh-Jeans law, eq. 4.4): $R_\oplus^2/R_\odot^2(T_\oplus/T_\odot)$ $10^{-4} \times 4 \times 10^{-2} = 4 \times 10^{-6}$.

(c) The fraction of the Sun's radiation reflected by Jupiter $\approx 3 \times 10^{-9}$. Ratio of Jupiter's luminosity to that of the Sun in thermal IR $\approx 2 \times 10^{-4}$.

Problem 14-13

$E \propto T$, $dE/dt \propto T^4$ (from eq. 4.11). Therefore, the radiative timescale $E/(dE/dt) \propto T^{-3} = 15^{-3} \approx 3 \times 10^{-4}$.

Problem 14-14

(a) The planet intercepts $\frac{10^{-3}}{1} \times \left(\frac{1}{0.03}\right)^2 \approx 1.11$ times as much energy per unit surface area as does the Earth.

Scale from Earth's equilibrium temperature of 255 K:

$$T_{\rm eq} = 255 \times \left[\frac{10^{-3}}{1} \times \left(\frac{1}{0.03}\right)^2\right]^{1/4} = 262\,\mathrm{K}.$$

So yes, this object does lie in the habitable zone if we assume that the planet has a similar albedo to its stars radiation as Earth and a modest greenhouse atmosphere like Earth's.

(b) The day side of this planet would continuous light from its star and be warmer than the planet average while the night side would never be heated by the star and be very cold. Regions near the terminator (boundary between day and night) and near the poles might have the most clement atmospheric conditions and be the "habitable zones" on such a planet.

(c) Atmospheric winds could carry energy from the dayside to the nightside thus somewhat equalizing the temperature differences. There would still likely be zones of habitable temperatures but they would likely be larger than in case **(b)**.

Problem 14-16

(a) $R_p = 3 \pm 1$ $\mathrm{R}_\oplus$, $M_p = 3 \pm 1$ $\mathrm{M}_\oplus$, so density is 200 – 3000 kg/m^3; the planet cannot be entirely rocky; if its density is above about 2000 kg/m^3, it could be primarily (compressed) water with a bit of rock; if it is low density it must have an H/He envelope that occupies a significant fraction of the volume; the planet's mass must be primarily elements heavier than He because a planet this small without heavy elements cannot be held together.

(b) $R_p = 1 \pm 0.5$ $\mathrm{R}_\oplus$, $M_p = 3 \pm 1$ $\mathrm{M}_\oplus$, so density 3000 – 80,000 kg/m^3, a very large range that includes large rocky planets with Earth-like composition, but probably denser with more iron; the lower density range would include lighter material as well as water; in contrast the denser portion of the range is unphysical, and indeed any value above around 20,000 kg/m^3 is unlikely because it requires a high abundance of extremely dense elements that are rare and aren't consistent with any other observations

(c) $R_p = 12 \pm 2$ $\mathrm{R}_\oplus$, $M_p = 300 \pm 100$ $\mathrm{M}_\oplus$, so this is a gas giant with most of the volume occupied by H/He; the measurements are not accurate enough to tell whether or not it contains a significant amount of heavier elements

(d) $R_p = 3 \pm 1$ $\mathrm{R}_\oplus$, $M_p = 30 \pm 10$ $\mathrm{M}_\oplus$, so density 2000 – 20,000 kg/m^3, i.e., not well constrained; probably H/He occupies most of the volume, but at the high density and small size end of the range it could be a (compressed) water-rich planet; by mass it must be mostly elements heavier than He in any case

(e) $R_p = 2 \pm 0.2$ $\mathrm{R}_\oplus$, $M_p = 10 \pm 1$ $\mathrm{M}_\oplus$, so density 5000 – 9000 kg/m^3, perhaps a super-earth with close to Earth-like composition, but likely with substantially more water or a bit of H/He

Problem 15-3

(a) Circular velocity:

$$v_{circ} = \sqrt{\frac{GM_\odot}{R_{1AU}}} \approx 30 \text{ km/s}.$$

By the virial theorem, the total mechanical energy for the molecule on a circular orbit is just equal to the negative of its kinetic energy:

$$E_{total} = -\frac{1}{2} m_{H_2} v_{circ}^2$$

$$m_{H_2} \approx 2 \times 1.673 \times 10^{-27} \text{ kg}$$

$$E_{total} \approx 3 \times 10^{-14} \text{ J}.$$

(b) Temperature increase:

$$H = c_p m_{H_2} \Delta T, \;\; c_p = 14340 \text{ J/kg K}$$

$$\Delta E_{heat} = \frac{1}{2} m_{H_2} v_{circ}^2 = c_p m_{H_2} \Delta T$$

$$\Delta T = \frac{v_{circ}^2}{2c_p} \approx 3.1 \times 10^4 \text{ K}.$$

Problem 15-4

(a) The gas in a protoplanetary disk is also in orbit, albeit with a velocity slightly slower than Keplerian because it is partly supported by a gradient in pressure.

$$n_{gas} \approx \sqrt{\frac{GM_\odot}{r^3}}(1-\eta), \;\; \eta \approx 5 \times 10^{-3}$$

$$\Delta v = \sqrt{\frac{GM_\odot}{R_{1AU}}} - n_{gas} \cdot R_{1AU}$$

$$\Delta v \approx 149 \text{ m/s}$$

$$V_{swept} = \Delta v \cdot (3.15 \times 10^7 \text{ s}) \cdot \pi R^2 \approx 4.7 \times 10^9 \cdot \pi R^2 \text{ m}$$

$$M_{swept} = \rho_{gas} \times V \approx 4700 \cdot \pi R^2 \text{ kg/yr}.$$

(b) Size of particle encountering its own mass in gas in one orbit:

$$\frac{4}{3}\pi R^3 \cdot \rho_{particle} = 4700 \cdot \pi R^2 \text{ kg}$$

$$R = \frac{4700 \cdot 3}{4 \cdot \rho_{particle}} \approx 1.17 \text{ m}.$$

Problem 15-6

For ordered growth $F_g = 10$ is constant, as is the rate of growth of the planetary radius, to first order...

$$\sigma_\rho = \frac{M_{Nep}}{\pi(35^2 - 25^2)(1.5 \times 10^{11})^2} \approx \frac{1.02 \times 10^{26}}{4.24 \times 10^{25}} \approx 2.4 \text{ kg/m}^2$$

and assuming $\rho_p =< \rho_{Nep} >= 1640$ kg/m^3,

$$\frac{dR}{dt} = \sqrt{\frac{3}{\pi}}\sigma_\rho \frac{\sqrt{\frac{GM_\odot}{30^3_{AU}}} \cdot 10}{4\rho_p} \approx 4.5 \times 10^{-12} \text{ m/s}$$

$$t_g \approx \left(\frac{dR}{dt}\right)^{-1} \times R_{Nep} = 5.4 \times 10^{18} \text{ s} \approx 1.7 \times 10^{11} \text{ years}$$

which is much longer than the Solar System age, so the process must not be ordered growth at or anywhere near Neptune's current location, especially since this provides only a rough lower limit.

Problem 15-7

Planets would be smaller, but there would be more of them. Growth would stop at smaller sizes because smaller mass planets could not perturb one another into crossing orbits. In the terrestrial planet region, the number of planets would increase to $\sqrt{2} \times 4 \approx 6$. Quantitative estimates are more difficult for the giant planets, but it is possible that there would be no Jupiter-like planets, thus several smaller planets and likely no asteroid belt.

Problem 15-12

Imagine a small volume V inside the center of the asteroid. The energy required to initiate melting in this volume is that required to raise its temperature up to the melting point from a starting point of, say, 100 K: $E_{\text{melt}} = 1700\,\text{K} \times V c_p \rho$. Assuming $\rho \sim 3000\,\text{kg m}^{-3}$ and $V = 1\,\text{m}^3$ this is about 4×10^9 J (for full melting, we would also need to include energy to overcome the latent heat of fusion).

The total energy produced by radioactive decay in our volume over a time τ is $E_{\text{tot}} = r_0 V \int_0^\tau e^{-tk} dt = (V r_0/k)[1 - e^{-\tau k}]$. If there were no loss of energy we could just equate $E_{\text{melt}} = E_{\text{tot}}$ and solve for τ. Neglecting energy loss for the moment (which is equivalent to considering a very large asteroid), enough energy is generated by radioactive decay at the initial rate to melt the material in 4×10^{12} seconds, or about 130,000 years. Accounting for the drop off in heat generation over time extends this a bit, but not much, since it is much shorter than the decay time of ^{26}Al.

We want the timescale for conduction to be longer than this radioactive heating timescale so that conduction is less important than energy production. The heat flux from conduction is given by (4.18): $Q = -K_{\text{T}} \nabla T$. Setting $\nabla T \sim -1700\,\text{K}/R$ and using the given value of K_{T} we know Q for a given R. Since we are interested in the smallest asteroid that can show some melting we can assume this heat flux is carried through a small cross sectional area encircling our volume V near the center, so the energy carried away by diffusion over time τ is $E_{\text{diff}} \sim 1\,\text{m}^2 \times \text{Q}\tau$. We can then just set $E_{\text{diff}}/E_{\text{melt}} < 1$ and solve for R using $\tau = 1/k$.

Problem 16-2

There are many reasons. A few key ones are:

1. Carbon has the unique ability to form long chains of complex molecules that have a high degree of stability. Stable complex molecules are required to build sugars, to build DNA, to build RNA, to build amino acids, proteins, cells, and finally, to build all living organisms on Earth.

2. Carbon can make 4 bonds, allowing for complex branching and linking into chains.

3. The C-H and C-C bands are of comparable strength.

4. Carbon is common, and even organic compounds are observed in many environments in the Universe, and this abundance makes them a good candidate for the building blocks of life.

Problem 16-3
(a) We can write the equilibrium temperature as

$$T_{\rm eq}^4 = \frac{1-A_{\rm b}}{16\sigma}\frac{L_\star}{\pi r^2}.$$

Solving for r in units of AU we have

$$\begin{aligned} r &= \left(\frac{1-A_{\rm b}}{16\pi\sigma}\frac{L_\star}{T_{\rm eq}^4}\right)^{1/2} \\ &\approx 77,200\left(\frac{(1-A_{\rm b})}{T_{\rm eq}^4}\frac{L_\star}{L_\odot}\right)^{1/2} \text{ AU.} \end{aligned}$$

We can then set $T_{\rm eq}$ to any value to solve for the equivalent r. For 273 and 373 K we find $r_{\rm AU} = 0.86$ and 0.46 AU.

(b) Venus is the only planet in the Sun's habitable zone by this definition.

(c) The greenhouse effect increases the surface temperature for a given $T_{\rm eq}$, thus moving the habitable zone outwards. There are many other neglected factors including including the characteristics of the planets orbit (highly eccentric?), rotation (planetary obliquity), composition (amount of oceans), latitudinal variations, etc. Many answers are possible.

Problem 16-9
If the Sun suddenly became twice as bright as it is at present, the Earth would intercept slightly more solar radiation than Venus does at present. Earth's temperature would increase substantially. More water would evaporate, increasing the greenhouse effect, eventually leading to a runaway greenhouse with an atmosphere that was very thick and composed primarily of water vapor. The oceans would evaporate and the surface temperature would rise to the melting point of rock. Hydrogen would escape to space, and eventually the Earth's water would be lost and the Earth would cool a bit to a state comparable to present-day Venus.

Problem 16-12

(a) $V_{spherule} = 4/3\ \pi R^3$

$R_{spherule} = 100\ \mu\text{m} = 0.1$ mm

$V_{spherule}$= 4,186,666 $\mu\text{m}^3 = 4.19 \times 10^{-3}\ \text{mm}^3$.

(b) Depends on how closely packed the layer is:

Rhombohedral packing is the most compact arrangement in space of uniform spheres (atoms and molecules in mineral crystals, or grains in sedimentary rocks) that results in a structure having no more than 26% porosity whereas the loosest packing, cubic packing, shows a porosity of 47%. Assuming rhombohedral packing with a porosity of 26%, 2.22 mm^3 of the 3 mm^3 would be occupied by spherules.

$\text{V}_{spherule} = 4.19 \times 10^{-3}\ \text{mm}^3$

Number of spherules = 530.

(c) cross-sectional area of a spherule

$$A_{spherule} = \pi R^2 = 3.14 \times 10^{-2}\text{mm}^2.$$

(d) $A^2_{spherules\ in\ 1mm} = A_{spherules} * X_{amount\ of\ spherules\ in\ 1mm^2}$

$A_{spherules\ in\ 1mm^2} = 3.14 \times 10^{-2}\text{mm}^2 \times 530 = 17\ \text{mm}^2$.

Assumptions:

Spherules are opaque

optical depth = $\frac{17\text{mm}^2}{1\text{mm}^2}$

Optical depth (around 17) means you have to remove 17 particles at any given point to see light coming through. The atmosphere after the impact at the K/T boundary was full of these small, opaque spherules and little light came through.

Problem 16-19

(a) We can write the equilibrium temperature as

$$T_{\text{eq}}^4 = \frac{1 - A_{\text{b}}}{16\pi\sigma}\frac{L}{R^2}.$$

Setting $R = 1\,\text{AU}$ and $L = L_\odot$ and $A_{\text{b}} = 0.29$ we have $T_{\text{eq}} = 255\,\text{K}$. Thus $288 - 255 = 33\,\text{K}$ is generated by the greenhouse effect.

(b) Now $T_{\text{eq}} = 255 \times \left(\frac{0.45}{0.71}\right)^{1/4} = 227\,\text{K}$. Adding in the same 33 K greenhouse effect brings the surface temperature to 260 K.

(c) If the global surface temperature ever fell well the freezing point, the ice caps would grow and the mean albedo would increase. With an increasing albedo, more sunlight gets reflected and the temperature drops. This process will finally lead to the formation of SnowBall Earth (SBE).

But even for SNE, the internal heat of Earth (due to radioactive decay, core solidification, etc.) will find its way out through volcanic activity, which injects CO_2 into the atmosphere. Since during the process of SBE almost no weathering is occurring, and the sequestration of of CO_2 would be extremely low as well, the atmosphere will continue to accumulate CO_2. When the atmospheric CO_2 abundance increases to a level that is enough to generate a sufficiently enhanced greenhouse effect, ice will start melting. Once the first ice is melted, the planet's albedo will decrease and the Earth could escape the snowball.