ENG 3PX3 - Engineering Economics

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Sensitivity Analysis

L03 Recap: Marginal Value Change

→The *marginal net value* of oranges is the extra net value obtained from one more orange:

$$\Delta NV_{Oranges}(x) = NV_{Oranges}(x + 1 \text{ orange}) - NV_{Oranges}(x)$$

→Considering the units of x (i.e., # of oranges), the marginal value change compared to the parameter change that produced it is the *current conversion rate* (of oranges into net value):

$$\frac{\Delta NV_{Oranges}}{\Delta Oranges}$$

→Alternatively,

$$NV'_{Oranges}(x) = \frac{dNV_{Oranges}}{dOranges}$$

Sensitivity Analysis Introduction

→**What's the change in NV when you receive from one extra orange?**

- **Marginal analysis** is an economic tool for determining the impact of a decision on net value, especially when the decision is incremental (e.g., change in NV with one more orange)
- **Sensitivity analysis** is a way to explore how sensitive the model (i.e., NVF) is to changes in its inputs or parameters (like conversion factors).
  - In other words, sensitivity analysis is like conducting marginal analysis for each variable separately and comparing the results (often visually, with a spider or tornado plot)

Recap: nanoRIMS Fluid Mechanics Tech Analysis – Impact

→Suppose technical analysis of pipe diameter leaves us (after modelling its impact on NVF) with a cost per week function for nanoRIMS as follows:

→Cost per week = Original cost per week + $\Delta Cst_{pump} + \Delta Cst_{\text{tube diameter uncertainty}}$

$$= \$202.04 + \frac{\$10}{52} \times \frac{\rho\dot{\mathcal{V}}\left(\frac{32\dot{\mathcal{V}}}{\pi D^4}\left(\frac{\dot{\mathcal{V}}}{\pi} + 12\nu L\right) + g\Delta z\right)}{10\ \text{mW}} + \$1.1875\,\frac{\pi D^2}{\text{cm}^2}$$

→(At least around pipe inner diameter of $D = 6.35\ \text{mm}$ and the following other parameter values:

- $\dot{\mathcal{V}} = 350$ mL/min $= 5.83\text{e}{-6}\ \text{m}^3/s$ is the volumetric flow rate,
- $\rho = 1000$ kg/m^3 and $\nu = 1\text{e}{-6}$ m$^2/s$ are the density and kinematic viscosity of water,
- $g = 9.81$ m/s^2 is the Earth's gravitational acceleration,
- $L = 30$ cm is the length of the tube,
- $\Delta z = 20$ cm is the (max) elevation change provided by the pump)

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- $L = 30$ cm is the length of the tube,
- $\Delta z = 20$ cm is the (max) elevation change provided by the pump)

How sensitive is the NV to changes in pipe diameter?

Recap: nanoRIMS Fluid Mechanics Tech Analysis – Impact

→The marginal rate of change of NV per change in pipe diameter at the input set of parameters is the partial derivative of NV wrt pipe diameter evaluated at that set of inputs.

→NV per week is benefits per week – cost per week = \$896/wk – cost per week, where:

$$\text{Cost per week} = \$202.04 + \frac{\$10}{52} \times \frac{\rho\dot{\mathcal{V}}\left(\frac{32\dot{\mathcal{V}}}{\pi D^4}\left(\frac{\dot{\mathcal{V}}}{\pi} + 12\nu L\right) + g\Delta z\right)}{10\text{ mW}} + \$1.1875\,\frac{\pi D^2}{\text{cm}^2}$$

→So,

$$\frac{\partial NV}{\partial D} = \frac{\partial}{\partial D}\left(+\frac{\$896}{\text{wk}} - \frac{\$202.04}{\text{wk}} - \frac{\$10}{52} \times \frac{\rho\dot{\mathcal{V}}\left(\frac{32\dot{\mathcal{V}}}{\pi D^4}\left(\frac{\dot{\mathcal{V}}}{\pi} + 12\nu L\right) + g\Delta z\right)}{10\text{ mW}} - \$1.1875\,\frac{\pi D^2}{\text{cm}^2}\right)$$

$$\frac{\partial NV}{\partial D} = -\frac{\$10}{52} \times \frac{\rho\dot{\mathcal{V}}\left(-4\frac{32\dot{\mathcal{V}}}{\pi D^5}\left(\frac{\dot{\mathcal{V}}}{\pi} + 12\nu L\right)\right)}{10\text{ mW}} - \$1.1875\,\frac{\pi 2D}{\text{cm}^2}$$

Recap: nanoRIMS Fluid Mechanics Tech Analysis – Impact

$$\frac{\partial NV}{\partial D} = -\frac{\$10}{52} \times \frac{\rho\dot{\mathcal{V}}\left(-4\frac{32\dot{\mathcal{V}}}{\pi D^5}\left(\frac{\dot{\mathcal{V}}}{\pi} + 12\nu L\right)\right)}{10\text{ mW}} - \$1.1875\frac{\pi 2D}{\text{cm}^2}$$

Substituting in the parameter values we'd then have the rate of change of NV with respect to pipe diameter at the specific set of input values (i.e., we'd know how *sensitive* net value is to pipe diameter at this point).

Sensitivity Analysis

→**To see how the model is affected by a specific input parameter, we can perform a sensitivity analysis**

- We change one parameter at a time, with all other things being equal (*Ceteris Paribus)*
  1. Determine the possible range of variation of one parameter,
  2. Evaluate the model outcome over this range
  3. Determine how (and whether) key conclusions change
  4. Select decisions that give the lowest and highest expected net value considering probabilities (if conclusions change).

Sensitivity Analysis: nanoRIMS

→How sensitive is the NV to changes in each of the parameters?

→Recall the NVF for nanoRIMS:

$$NV = \frac{\$896}{\text{week}} - \left(C_{ingred} + C_{space} + C_{time} + C_{device}\right)$$

→After technical analysis we included the cost of pump and tubing uncertainty

$$NV = \frac{\$896}{\text{week}} - \left(\frac{\$5}{100\text{ mL}} \times q_{ingred} + \frac{\$12.5}{\text{hr}} \times \frac{1}{8} \times t_{FumeHood} + \frac{\$15}{\text{hr}} \times t_{GradStudent} + C_{device} + \frac{\$10}{52} \times \frac{\rho\dot{\mathcal{V}}\left(\frac{32\dot{\mathcal{V}}}{\pi D^4}\left(\frac{\dot{\mathcal{V}}}{\pi} + 12\nu L\right) + g\Delta z\right)}{10\ \mathbf{mW}} + \$1.1875\,\frac{\pi D^2}{\mathbf{cm}^2}\right.$$

→How many parameters are there?

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Sensitivity Analysis: nanoRIMS

$$NV = \frac{\$896}{\text{week}} - \left(\frac{\$5}{100\ \text{mL}} \times q_{ingred} + \frac{\$12.5}{\text{hr}} \times \frac{1}{8} \times t_{FumeHood} + \frac{\$15}{\text{hr}} \times t_{GradStudent} + C_{device} + \frac{\$10}{52} \times \frac{\rho\dot{\mathcal{V}}\left(\frac{32\dot{\mathcal{V}}}{\pi D^4}\left(\frac{\dot{\mathcal{V}}}{\pi} + 12\nu L\right) + g\Delta z\right)}{10\ \mathbf{mW}} + \$1.1875 \frac{\pi D^2}{\mathbf{cm}^2} \right)$$

→**How many parameters are there?**

- This equation has the following variables q_{ingred}, $t_{FumeHood}$, $t_{GradStudent}$, C_{device}, D, ρ, $\dot{\mathcal{V}}$, ν, L, g, Δz.
  - There are also other terms that are hiding because we've substituted them in: $\frac{\$12.5}{\text{hr}}$ is the cost of occupying the entire fume hood, $\frac{1}{8}$ is the fraction of the fume hood we're estimating nanoRIMS will take up, etc.
- You might perform a sensitivity analysis on this for different reasons:
  - On a decision variable like D (water tube's internal diameter), sensitivity analysis tells you whether you should increase or decrease the variable, other things equal.
  - On a parameter we don't choose like g (gravitational acceleration near the Earth), sensitivity analysis tells you how important it is to determine it more precisely (if off by 1%, how big of a deal is this? How much can we trust the result?)

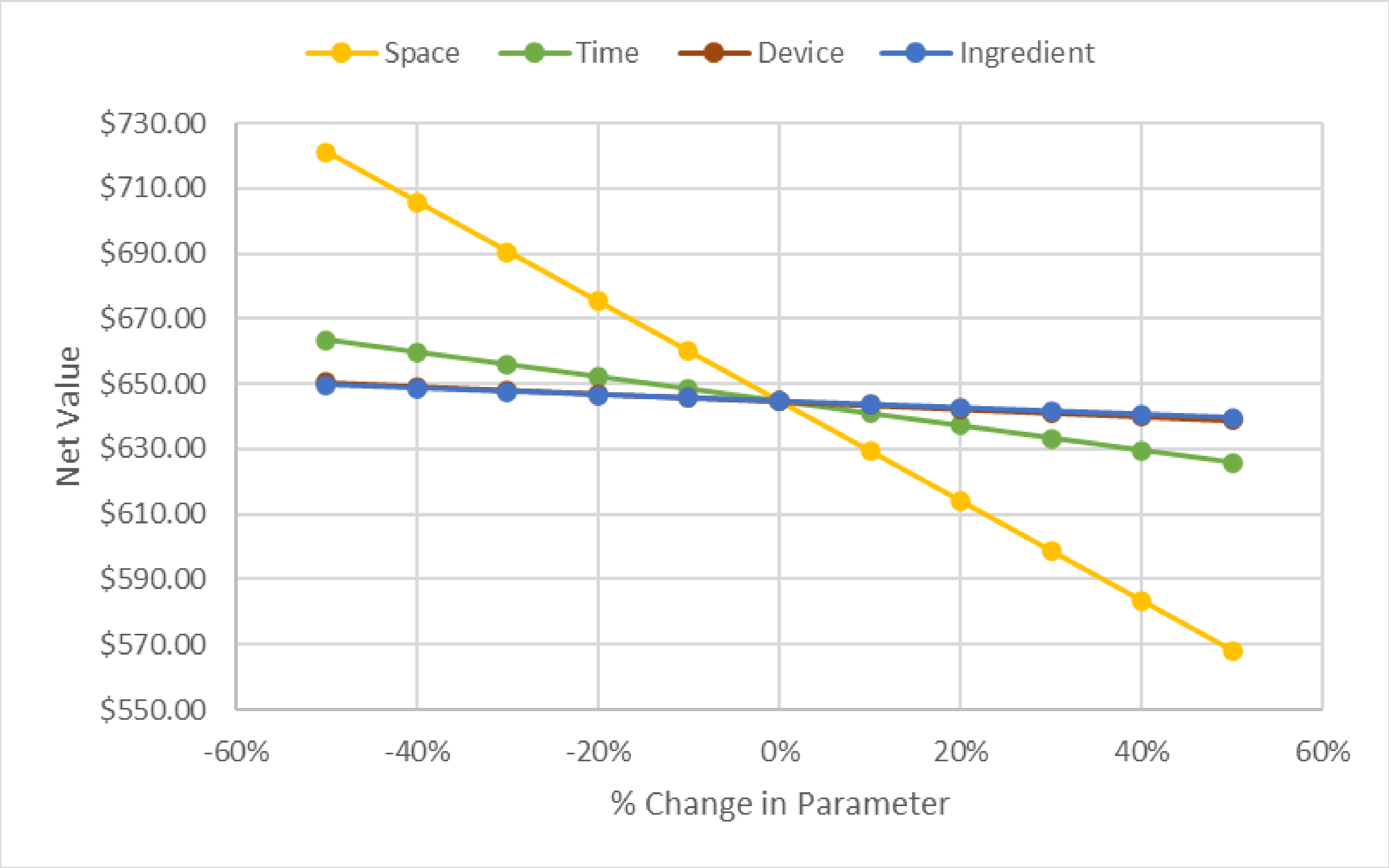
Sensitivity Analysis Visualization

→We can visualize the sensitivity of the NVF to certain parameters by using:

- A spider plot
- A tornado plot

→Multiple NVF parameters can be plotted on the same graph to compare their effect on NV and determine the parameters it's most sensitive to

Spider Plot



Space
Time
Device
Ingredient
$730.00
$710.00
$690.00
$670.00
$650.00
$630.00
$610.00
$590.00
$570.00
$550.00
Net Value
-60%
-40%
-20%
0%
20%
40%
60%
% Change in Parameter

Spider Plot in Excel

→**Determine the base case for your NVF**

→**Choose one NVF parameter to change**

- Change the parameter by x% (e.g. -20%, 20% etc.)
- In a table recalculate NV after each time you have changed the parameter, keeping all other parameters at the base case.

→**Repeat for another NVF parameter to change (keeping all others, including the first one you changed, at the base case)**

→**Repeat steps above until you have investigated all the parameters you want to investigate**

Spider Plot Example: Exploring a decision Variable

pi	3.14	
V (flow rat	5.83E-06	m3/s
ρ	1000	kg/m3
v (kinemat	1.00E-06	m2/s
g	9.81	m/s2
L	3.00E-01	m
z	0.2	m
D	6.35E-03	m
W	1.96E+00	W

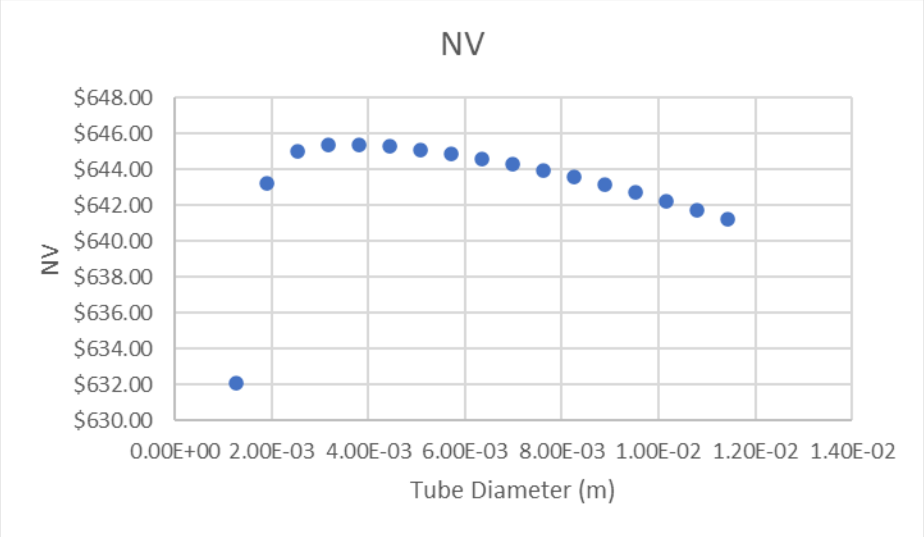
nanoRIMS				
			per week needed	
Ingredients	$5	/100 mL	2	100 mL
Fume Hood Time	$1.56	/hr	98	hr
Device	$600	/yr	0.019231	
Grad Student Time	$15	/hr	2.5	hr
Cost pump	$38			
Cost tube uncertaint	$2			
NV (rel to Purchasing	$644.58	/week		

Optimizing Decision Variable

nanoRIMS				
			per week needed	
Ingredient	$5	/100 mL	2	100 mL
Fume Hoo	$1.56	/hr	98	hr
Device	$600	/yr	0.019231	
Grad Stud	$15	/hr	2.5	hr
Cost pump	$38			
Cost tube	$1.50			
NV (rel to	$644.58	/week		

D	6.35E-03	m
W	1.96E+00	W

%change	D	W	Cpump	Cuncertair	NVF
-80	1.27E-03	2.69E+00	5.17E+01	6.01E-02	$632.07
-70	1.91E-03	2.11E+00	4.05E+01	1.35E-01	$643.21
-60	2.54E-03	2.01E+00	3.86E+01	2.41E-01	$644.99
-50	3.18E-03	1.98E+00	3.81E+0		
-40	3.81E-03	1.97E+00	3.79E+0		
-30	4.45E-03	1.97E+00	3.78E+0		
-20	5.08E-03	1.96E+00	3.78E+0		
-10	5.72E-03	1.96E+00	3.78E+0		
0	6.35E-03	1.96E+00	3.78E+0		
10	6.99E-03	1.96E+00	3.77E+0		
20	7.62E-03	1.96E+00	3.77E+0		
30	8.26E-03	1.96E+00	3.77E+0		
40	8.89E-03	1.96E+00	3.77E+0		
50	9.53E-03	1.96E+00	3.77E+0		
60	1.02E-02	1.96E+00	3.77E+0		
70	1.08E-02	1.96E+00	3.77E+01	4.35E+00	$641.76
80	1.14E-02	1.96E+00	3.77E+01	4.87E+00	$641.23



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W	1.96E+00	W

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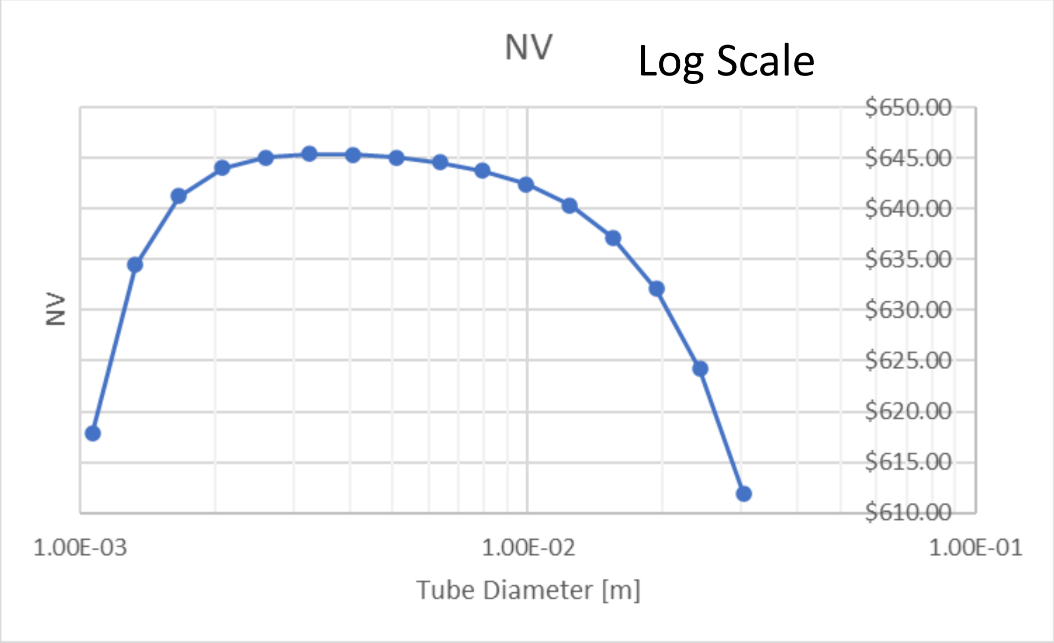
D	6.35E-03	m
W	1.96E+00	W

D	W	Cpump	Cuncertain	NVF
1.07E-03	3.43E+00	6.59E+01	4.23E-02	$617.85
1.33E-03	2.56E+00	4.93E+01	[illegible]	[illegible]
1.66E-03	2.21E+00	4.25E+01	[illegible]	[illegible]
2.08E-03	2.06E+00	3.97E+01	[illegible]	[illegible]
2.60E-03	2.00E+00	3.85E+01	[illegible]	[illegible]
3.25E-03	1.98E+00	3.81E+01	[illegible]	[illegible]
4.06E-03	1.97E+00	3.79E+01	[illegible]	[illegible]
5.08E-03	1.96E+00	3.78E+01	[illegible]	[illegible]
6.35E-03	1.96E+00	3.78E+01	[illegible]	[illegible]
7.94E-03	1.96E+00	3.77E+01	[illegible]	[illegible]
9.92E-03	1.96E+00	3.77E+01	[illegible]	[illegible]
1.24E-02	1.96E+00	3.77E+01	[illegible]	[illegible]
1.55E-02	1.96E+00	3.77E+01	[illegible]	[illegible]
1.94E-02	1.96E+00	3.77E+01	[illegible]	[illegible]
2.42E-02	1.96E+00	3.77E+01	[illegible]	[illegible]
3.03E-02	1.96E+00	3.77E+01	[illegible]	[illegible]

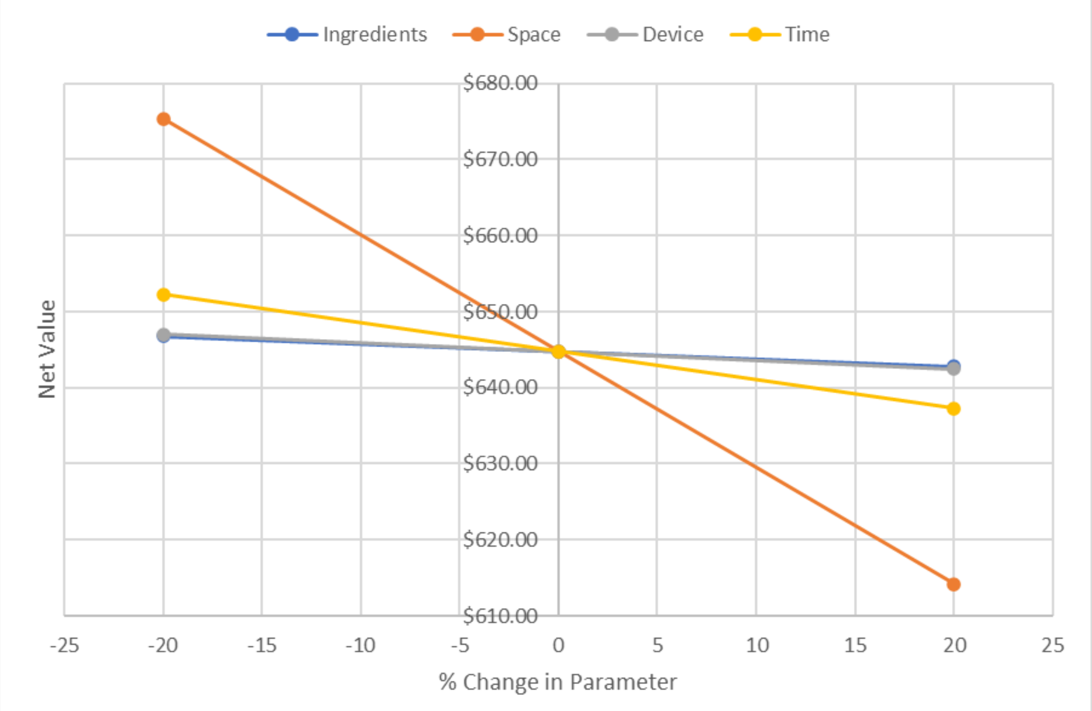


Spider Plot Example: Changing Parameter Values

→Simple Version: -20% & +20% for each parameter:

Sensitivity Analysis											
Ingredients Cost				Space Occupied				Device Cost			
Cost	% Change	NV	%Change in NV	Cost	% Change	NV	%Change in NV	Cost	% Change	NV	%Change in NV
6	20	$642.77	-0.31%	1.875	20	$614.15	-4.75%	$708.00	20	$642.50	-0.35%
$5	0	$644.77	0.00%	$1.56	0	$644.77	0.00%	$590.00	0	$644.77	0.00%
4	-20	$646.77	0.31%	1.25	-20	$675.40	4.75%	472	-20	$647.04	0.35%

GS Time			
Cost	% Change	NV	%Change in NV
18	20	$637.27	-1.16%
$15	0	$644.77	0.00%
12	-20	$652.27	1.16%



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Example Spider Plot Table

→Detailed Version: 11 points from -50% to +50% -> Generate data automatically using What-If Analysis!

		Base case	-50%	-40%	-30%	-20%	-10%	0%	10%	20%	30%	40%	50%
Ingredient	Cost	$5.00	$2.50	$3.00	$3.50	$4.00	$4.50	$5.00	$5.50	$6.00	$6.50	$7.00	$7.50
	NV	$644.58	$649.58	$648.58	$647.58	$646.58	$645.58	$644.58	$643.58	$642.58	$641.58	$640.58	$639.58
Space	Cost	$1.56	$0.78	$0.94	$1.09	$1.25	$1.41	$1.56	$1.72	$1.88	$2.03	$2.19	$2.34
	NV	$644.58	$721.14	$705.83	$690.52	$675.20	$659.89	$644.58	$629.27	$613.95	$598.64	$583.33	$568.02
Time	Cost	$15.00	$7.50	$9.00	$10.50	$12.00	$13.50	$15.00	$16.50	$18.00	$19.50	$21.00	$22.50
	NV	$644.58	$663.33	$659.58	$655.83	$652.08	$648.33	$644.58	$640.83	$637.08	$633.33	$629.58	$625.83
Device	Cost	$600.00	$300.00	$360.00	$420.00	$480.00	$540.00	$600.00	$660.00	$720.00	$780.00	$840.00	$900.00
	NV	$644.58	$650.35	$649.20	$648.04	$646.89	$645.73	$644.58	$643.43	$642.27	$641.12	$639.96	$638.81

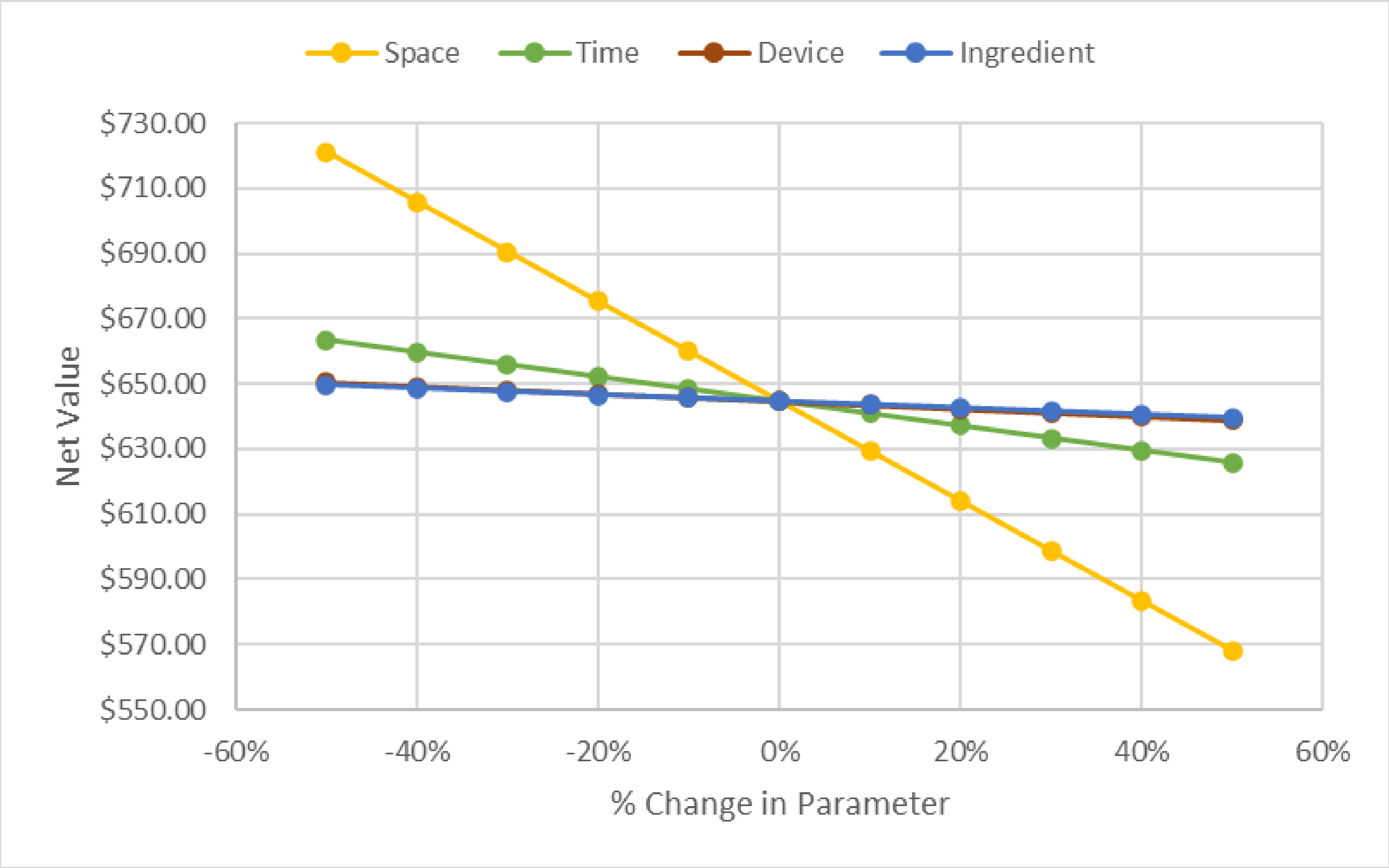
Spider Plot in Excel

→Optional: You can center the NVs to the base case by subtracting the calculated NV by the base case NV.

→For each parameter, plot the calculated NVs as a function of % Parameter Change

→Insert a legend

Spider Plot



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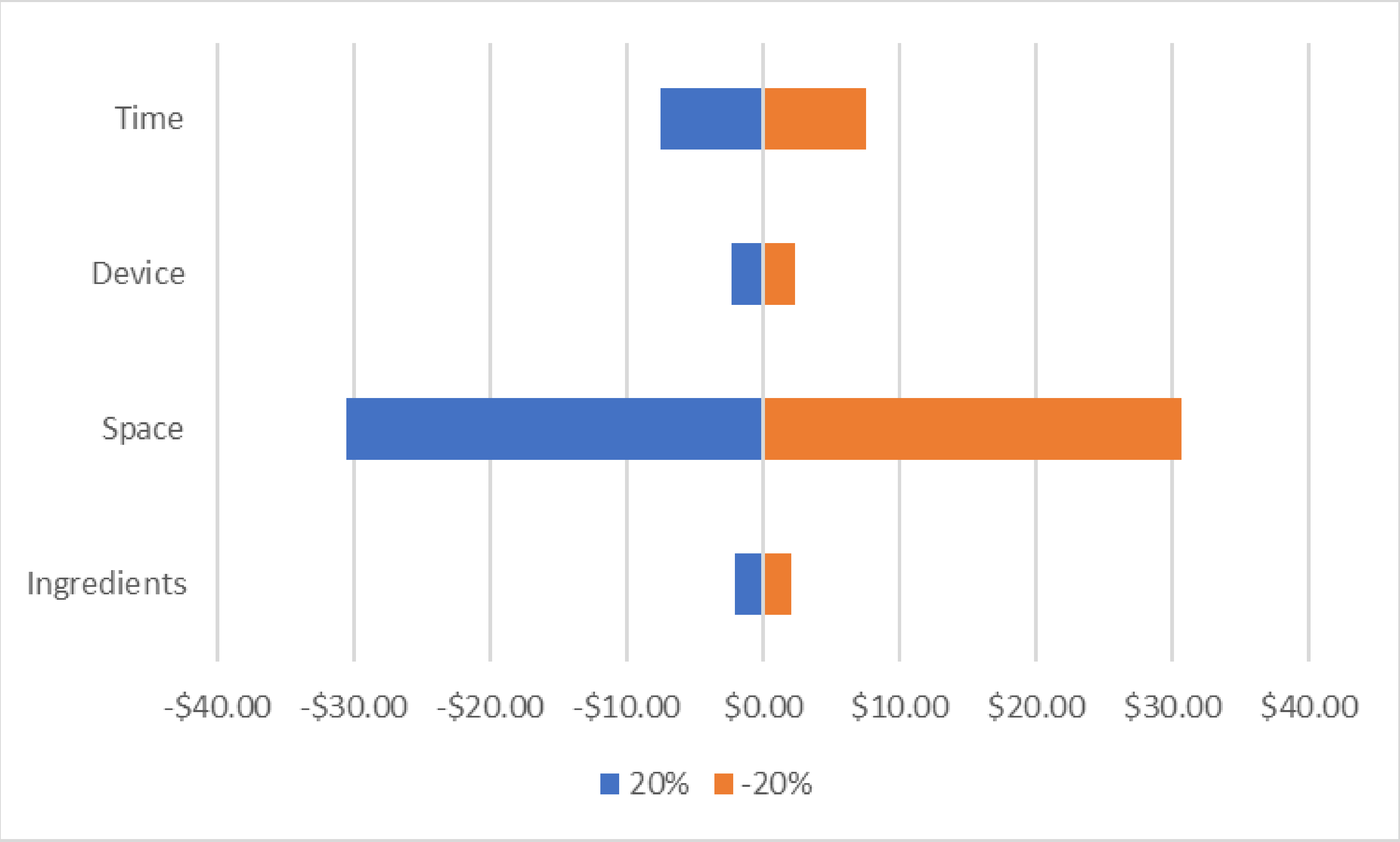
Tornado Plot



Tornado Plot in Excel

→Determine the base case for your NVF

→Choose one NVF parameter to change

→Change the parameter by x% and –x% (e.g. 20%, -20%)

→In a table calculate NV with the change in parameter

→Return NVF to base case

→Choose another NVF parameter to change

→Repeat steps above until you have investigated all the parameters you want to investigate

Tornado Plot in Excel

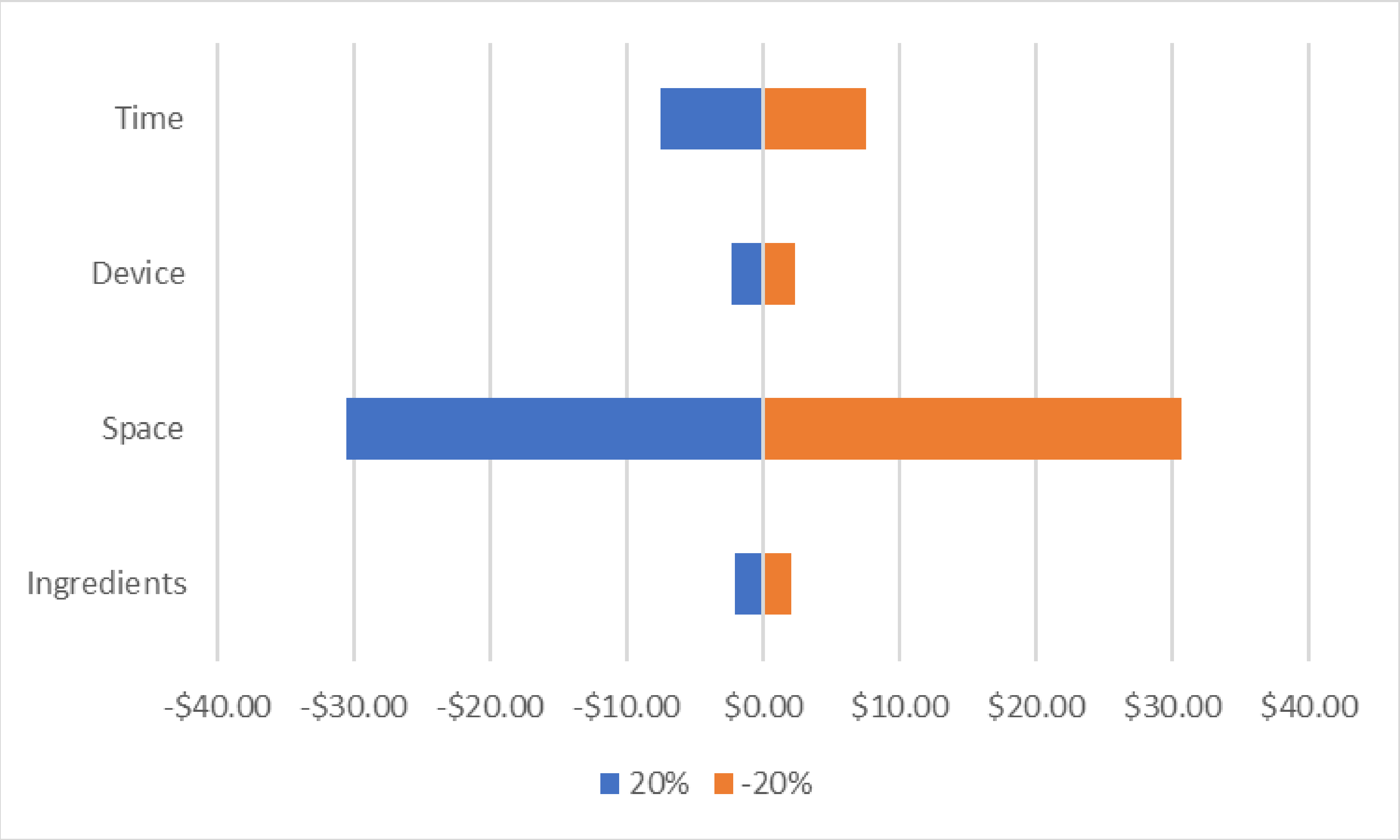
→Center the NVs to the base case by subtracting the calculated NV by the base case NV.

→Plot the centered NVs as a function of % Parameter Change in a stacked bar plot

→Insert a legend

	-20%	20%	-20%	20%
Ingredients	$646.58	$642.58	$2.00	-$2.00
Space	$675.20	$613.95	$30.63	-$30.63
Device	$646.89	$642.27	$2.31	-$2.31
Time	$652.08	$637.08	$7.50	-$7.50

Tornado Plot



Sensitivity Analysis

- →From these results, we can say that the NV is most sensitive to changes in space occupied (that is, a 1% change in space occupied makes more difference than a 1% change in the other parameters we explored, at least at the current values for the parameters)
- →If the model gets more complicated (i.e., more NVF parameters), we may remove ingredient cost as a parameter if we deem it to have an insignificant effect on the NV
- →We can use sensitivity analysis to make more informed decisions that will help us maximize the NV most efficiently

Sensitivity Analysis Visualization

→**There are two key drawbacks of sensitivity graphs**

1. Parameter interactivity: only changes one parameter at a time, so doesn't capture *interactions* between parameters
2. Locality: Results are limited to a specific range, outside this range parameters may not behave in a similar way

Places you'll use Sensitivity Analysis in the 3PX Project

→**Prior to optimization, for the default set of decision variables:**

- Explore impact of changing your decision variables on NV
  - will help suggest method of optimizing
- Explore impact of changing other parameters (e.g., conversion factors) on NV
  - will inform which are most important to get right, and how much we can trust the results.

→**After optimization, for the optimum set of decision variables:**

- Explore impact of changing other parameters (e.g., conversion factors)
  - on NV
  - on which set of decision variable values is optimum (with help of Excel sensitivity report)