ENG 3PX3 - Engineering Economics

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Linear Optimization

Linear Optimization

→**Linear programming (i.e., linear optimization) is the best (i.e., optimal) method of optimizing if the model is linear**

- Even when the model is nonlinear (i.e., contains nonlinear terms such as $\sqrt{\boldsymbol{x}}, \boldsymbol{x}^{\boldsymbol{2}}, \boldsymbol{x}_{\boldsymbol{1}}\boldsymbol{x}_{\boldsymbol{2}},$ ...), you might want to *locally linearize it** just to use linear programming.
  - *You can do this with a linearization (a first order Taylor series expansion) around the point

→**Wide variety of use cases in economics and beyond**

- Resource allocation
- Project selection
- Scheduling and Capital budgeting
- Energy network optimization

→**Is the most widely used optimization method (by people; *maybe* not machines or nature)**

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Linear Program Definition

→**An optimization model is a Linear Program (and therefore can be solved with *linear programming*) if it:**

- is comprised of only continuous variables (variables aren't discrete or discontinuous),
- has a single linear objective function (e.g., $3x + 2y - z$ instead of $3x^2 + 2y - z$, otherwise could have a max or min *not* at a boundary), and
- has only linear equality and/or inequality constraints (i.e., boundaries of the feasible region are straight lines).

Linear Program Definition

→A linear program has the following general formulation notation:

$$\min_{\boldsymbol{x}} \phi = \boldsymbol{c}^T \boldsymbol{x}$$ ← **Objective Function**

s.t. ← "Subject to"

$$A_h \boldsymbol{x} = \boldsymbol{b}_h$$ ← Equality Constraints

$$A_g \boldsymbol{x} \leq \boldsymbol{b}_g$$ ← Inequality Constraints

$$\boldsymbol{x}_{lb} \leq \boldsymbol{x} \leq \boldsymbol{x}_{ub}$$ ← Variable Bounds

- x_j → j^{th} decision variable.
- c_j → j^{th} cost coefficient for the j^{th} decision variable.
- $a_{i,j}$ → constraint coefficient for variable j in constraint i.
- b_i → RHS coefficient for constraint i
- A_h and A_g both have n columns and m_h and m_g rows, respectively

Linear Program Definition

→**Note how this notion is different from the 'general' formulation previously described**

Generalized Program

$$\min_{\boldsymbol{x}} \phi = f(\boldsymbol{x})$$

s.t.

$$h(\boldsymbol{x}) = 0$$

$$g(\boldsymbol{x}) \leq 0$$

$$\boldsymbol{x}_{lb} \leq \boldsymbol{x} \leq \boldsymbol{x}_{ub}$$

Linear Program

$$\min_{\boldsymbol{x}} \phi = \boldsymbol{c}^T \boldsymbol{x}$$

s.t.

$$A_h \boldsymbol{x} = \boldsymbol{b}_h$$

$$A_g \boldsymbol{x} \leq \boldsymbol{b}_g$$

$$\boldsymbol{x}_{lb} \leq \boldsymbol{x} \leq \boldsymbol{x}_{ub}$$

→We can replace the objective function $f(\boldsymbol{x})$ with a left-multiplication by vector $\boldsymbol{c}^T$ (i.e., a dot product) because we specify that the objective function is a linear combination of the decision variables $\boldsymbol{x}$

→The constraints are updated to specify linear combinations instead of general functions

Recall: Linear Optimization Example

→ **A refinery distills crude petroleum into three products: gasoline, jet fuel, and lubricants.**

→ **Your plant receives crude oil shipments from two locations: Canada and USA**

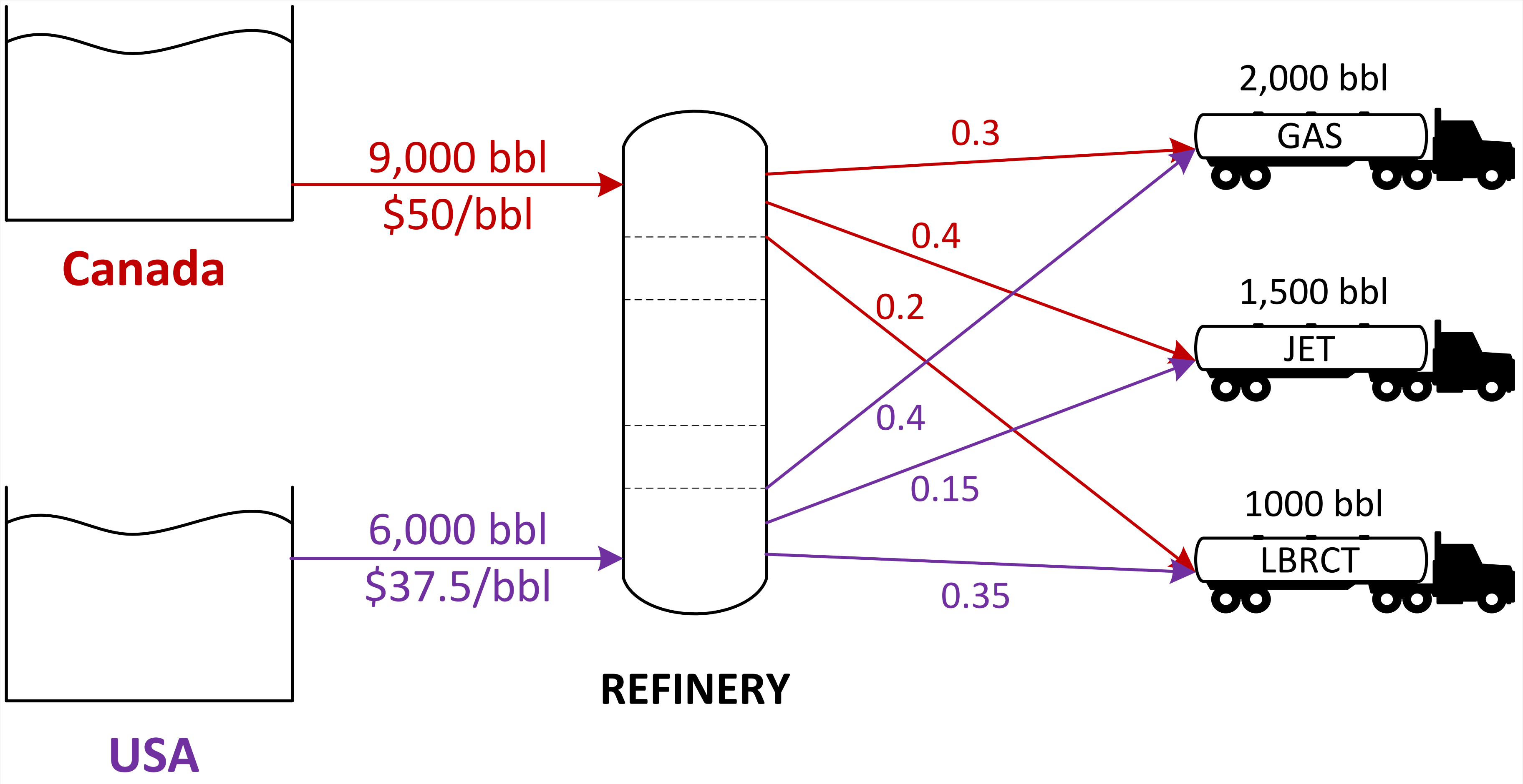
- Each barrel from Canada yields 0.3 barrels of gasoline, 0.4 barrels of jet fuel, and 0.2 barrels of lubricants.
- Each barrel from USA yields 0.4 barrels of gasoline, 0.15 barrels of jet fuel, and 0.35 barrels of lubricants.
- The remaining 0.1 (10%) from both sources is lost to the refining process.
- The Canadian oil costs your refinery $50 per barrel and is available up to 9,000 barrels per day.
- The American crude costs your refinery $37.5 but only available up to 6,000 barrels per day.

→ **You have a contract with local distributors to provide per day**

- 2,000 barrels of gasoline
- 1,500 barrels of jet fuel
- 1000 barrels of lubricant

Linear Optimization Example

→Draw a diagram of the supply network



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Recall: Linear Optimization Example

→Formulate the linear optimization problem:

$$
\begin{array}{rcll}
\min\limits_{x_1, x_2} \phi & = & 50x_1 + 37.5x_2 & \\
\multicolumn{4}{c}{\text{Subject to}} \\
0.3x_1 + 0.4x_2 & \geq & 2,000 & \\
0.4x_1 + 0.15x_2 & \geq & 1,500 & \\
0.2x_1 + 0.35x_2 & \geq & 1,000 & \\
x_1 & \leq & 9,000 & \\
x_2 & \leq & 6,000 & \\
x_i & \geq & 0 & (\forall\, i)
\end{array}
$$

Recall: Linear Optimization Example - Excel

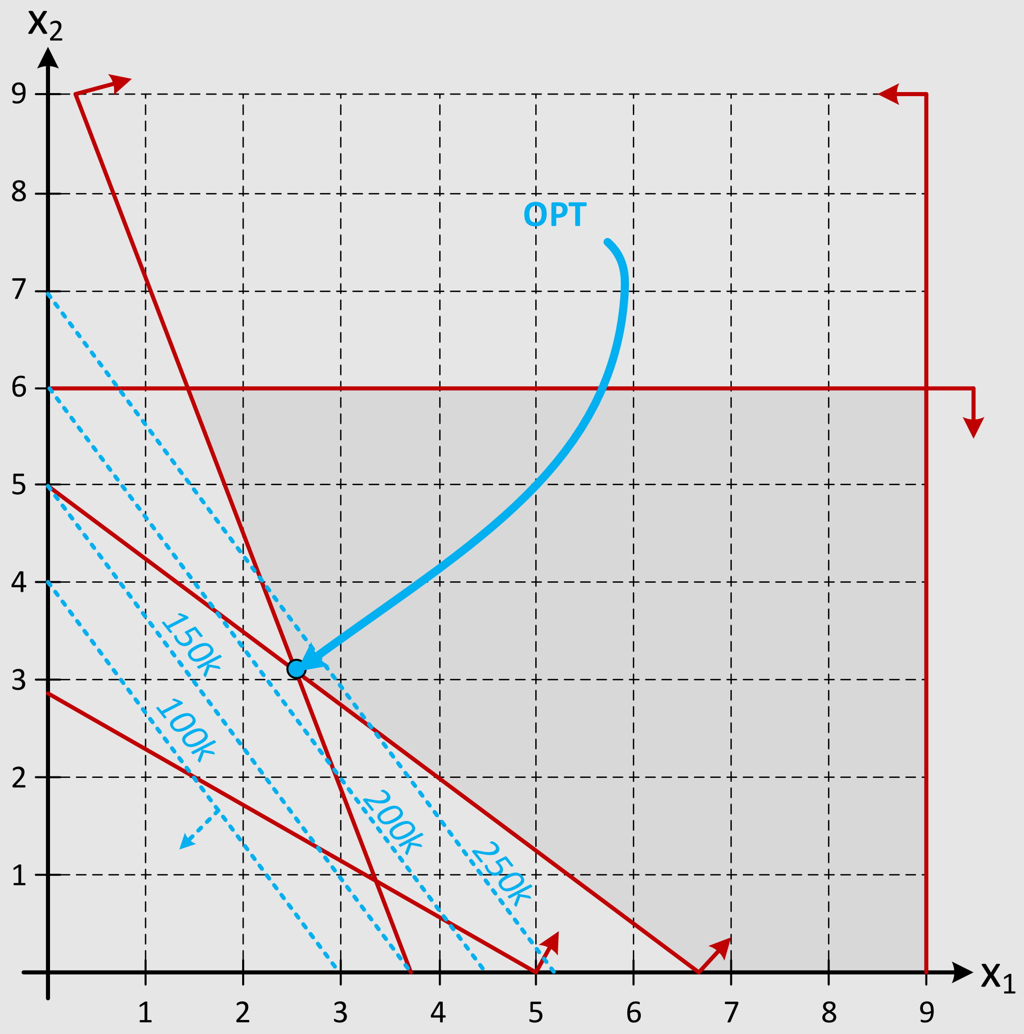
→For this simple linear model, we can start with a graphical model:

- Plot the constraints and objective function
- Visually identify the optimal value and feasible set

$$\min_{x_1,x_2} \phi = 50x_1 + 37.5x_2$$

Subject to

$$0.3x_1 + 0.4x_2 \geq 2,000$$
$$0.4x_1 + 0.15x_2 \geq 1,500$$
$$0.2x_1 + 0.35x_2 \geq 1,000$$
$$x_1 \leq 9,000$$
$$x_2 \leq 6,000$$
$$x_i \geq 0 \quad (\forall\, i)$$



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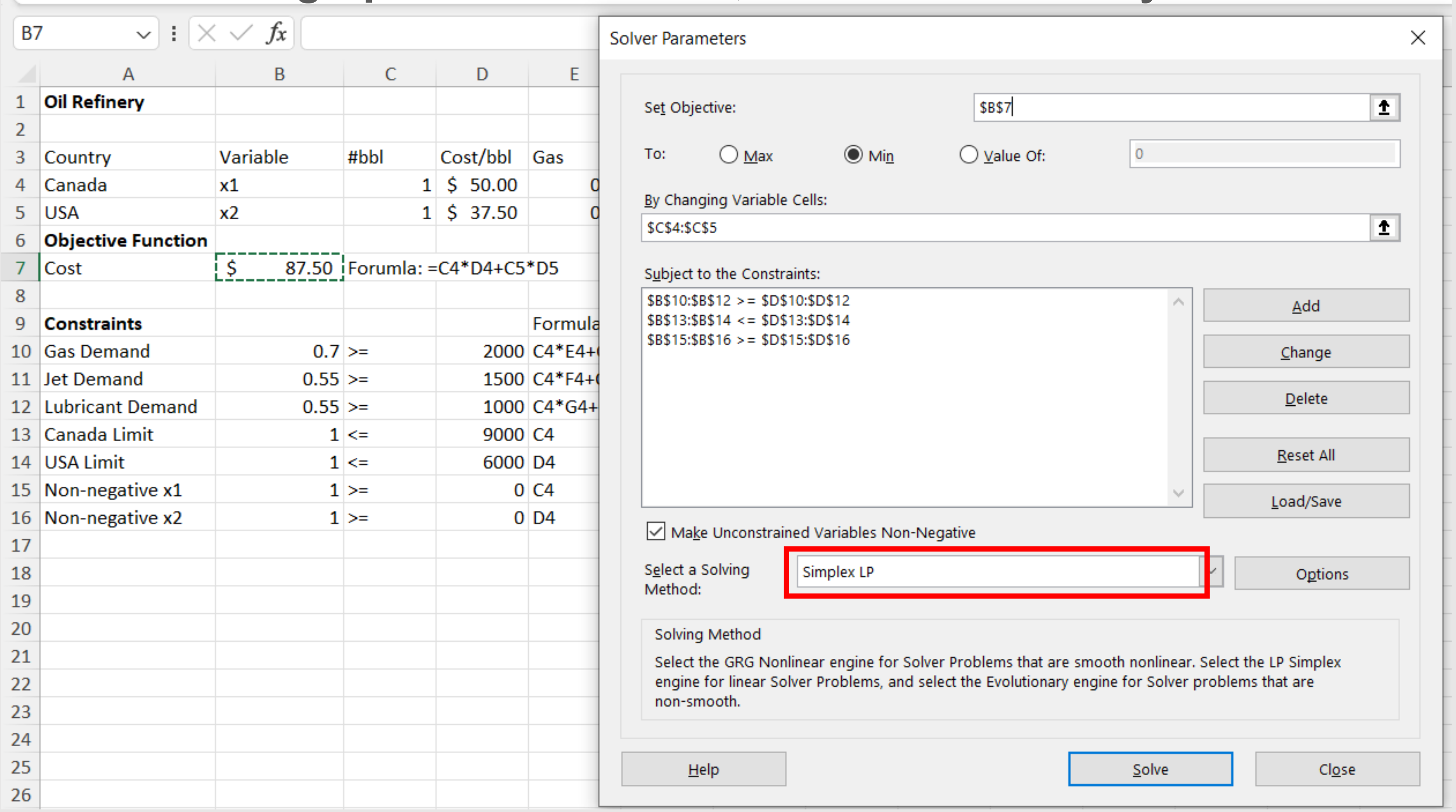
Linear Optimization Example - Excel

- →We can also verify this solution using Excel
- →Our model can be entered into Excel like this →

	A	B	C	D	E	F	G
1	**Oil Refinery**						
2							
3	Country	Variable	#bbl	Cost/bbl	Gas	Jet	Lubricant
4	Canada	x1	1	$ 50.00	0.3	0.4	0.2
5	USA	x2	1	$ 37.50	0.4	0.15	0.35
6	**Objective Function**						
7	Cost	$ 87.50	Forumla: =C4*D4+C5*D5				
8							
9	**Constraints**				Formula		
10	Gas Demand	0.7	>=	2000	C4*E4+C5*E5		
11	Jet Demand	0.55	>=	1500	C4*F4+C5*F5		
12	Lubricant Demand	0.55	>=	1000	C4*G4+C5*G5		
13	Canada Limit	1	<=	9000	C4		
14	USA Limit	1	<=	6000	D4		
15	Non-negative x1	1	>=	0	C4		
16	Non-negative x2	1	>=	0	D4		

Linear Optimization Example - Excel

→After setting up the Excel sheet, use Data → Analyze → Solver:



For linear models use “Simplex LP”

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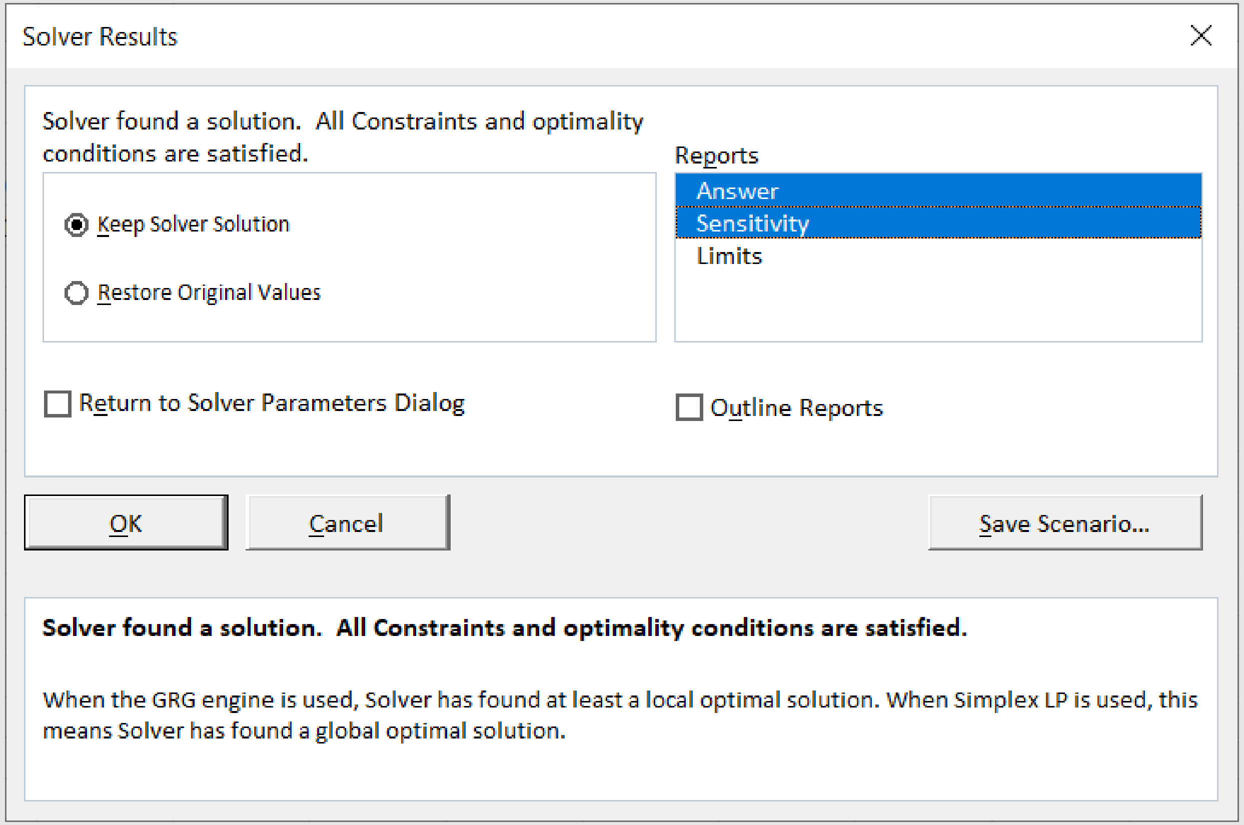
Linear Optimization Example - Excel

→Solve → OK



	A	B	C	D	E	F	G
1	**Oil Refinery**						
2							
3	Country	Variable	#bbl	Cost/bbl	Gas	Jet	Lubricant
4	Canada	x1	2608.696	$ 50.00	0.3	0.4	0.2
5	USA	x2	3043.478	$ 37.50	0.4	0.15	0.35
6	**Objective Function**						
7	Cost	$ 244,565.22	Forumla: =C4*D4+C5*D5				
8							
9	**Constraints**				Formula		
10	Gas Demand	2000	>=	2000	C4*E4+C5*E5		
11	Jet Demand	1500	>=	1500	C4*F4+C5*F5		
12	Lubricant Demand	1586.956522	>=	1000	C4*G4+C5*G5		
13	Canada Limit	2608.695652	<=	9000	C4		
14	USA Limit	3043.478261	<=	6000	D4		
15	Non-negative x1	2608.695652	>=	0	C4		
16	Non-negative x2	3043.478261	>=	0	D4		

Answer Report

→**The Answer Report gives the following computation details:**

- Time and number of iterations to solve
- Objective function results
- Decision variables
- Constraints (cell value, formula, status (binding/not binding) and slack)

→***Slack***

- shows how far the optimal solution is from the constraint.
- For binding constraints, the slack is 0. Changing non-binding variables within their slack value will not change the optimal solution.

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$B$7	Cost Variable	$ 87.50	$ 244,565.22

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$4	Canada #bbl	1	2608.695652	Contin
$C$5	USA #bbl	1	3043.478261	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$10	Gas Demand Variable	2000	$B$10>=$D$10	Binding	0
$B$11	Jet Demand Variable	1500	$B$11>=$D$11	Binding	0
$B$12	Lubricant Demand Variable	1586.956522	$B$12>=$D$12	Not Binding	586.9565217
$B$13	Canada Limit Variable	2608.695652	$B$13<=$D$13	Not Binding	6391.304348
$B$14	USA Limit Variable	3043.478261	$B$14<=$D$14	Not Binding	2956.521739
$B$15	Non-negative x1 Variable	2608.695652	$B$15>=$D$15	Not Binding	2608.695652
$B$16	Non-negative x2 Variable	3043.478261	$B$16>=$D$16	Not Binding	3043.478261

Sensitivity Report (1/2): Decision Variables

→The Sensitivity Report is used to analyze how the model's constraints affect the optimum

→Decision variables (final value, reduced cost, objective coefficient (c), allowable increase/decrease)

- **Reduced cost** is the amount the objective function will change if the variable bounds are tightened ($\boldsymbol{x}_{lb}$ is increased or $\boldsymbol{x}_{ub}$ is decreased)
- **Allowable increase/decrease** indicate how much the objective coefficient must change before the optimal solution changes (note that the objective function value may change, but not the solution)

→**100% Rule:** If there are simultaneous changes to the objective coefficients and

$$\sum_{each\ coefficient}\left(\frac{Proposed\ Change}{Allowable\ Change}\right) \leq 100\%$$

then the optimal solution would not change

	A	B	C	D	E	F	G	H
4								
5								
6	Variable Cells							
7				Final	Reduced	Objective	Allowable	Allowable
8		Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9		$C$4	Canada #bbl	2608.695652	0	50	50	21.875
10		$C$5	USA #bbl	3043.478261	0	37.5	29.16666667	18.75
11								
12	Constraints							
13				Final	Shadow	Constraint	Allowable	Allowable
14		Cell	Name	Value	Price	R.H. Side	Increase	Decrease
15		$B$10	Gas Demand Variable	2000	65.2173913	2000	850	613.6363636
16		$B$11	Jet Demand Variable	1500	76.08695652	1500	1166.666667	750
17		$B$12	Lubricant Demand Variable	1586.956522	0	1000	586.9565217	1E+30
18		$B$13	Canada Limit Variable	2608.695652	0	9000	1E+30	6391.304348
19		$B$14	USA Limit Variable	3043.478261	0	6000	1E+30	2956.521739
20		$B$15	Non-negative x1 Variable	2608.695652	0	0	2608.695652	1E+30
21		$B$16	Non-negative x2 Variable	3043.478261	0	0	3043.478261	1E+30

Sensitivity Report (2/2): Constraints

→Constraints (final value, shadow price, constraints RH side, allowable increase/decrease)

- **Final value** shows the value of the constraints at the optimal solution
  - The difference between Constraints R.H. side and the final value is the slack of the constraint
- The **shadow price** of a constraint is the marginal improvement of the objective function value if the RHS is increased by 1 unit (while holding all other constraints constant)
  - All inactive constraints will have a shadow price of 0
- **Allowable increase/decrease** shows how much the constraint can change before the shadow price changes

Variable Cells

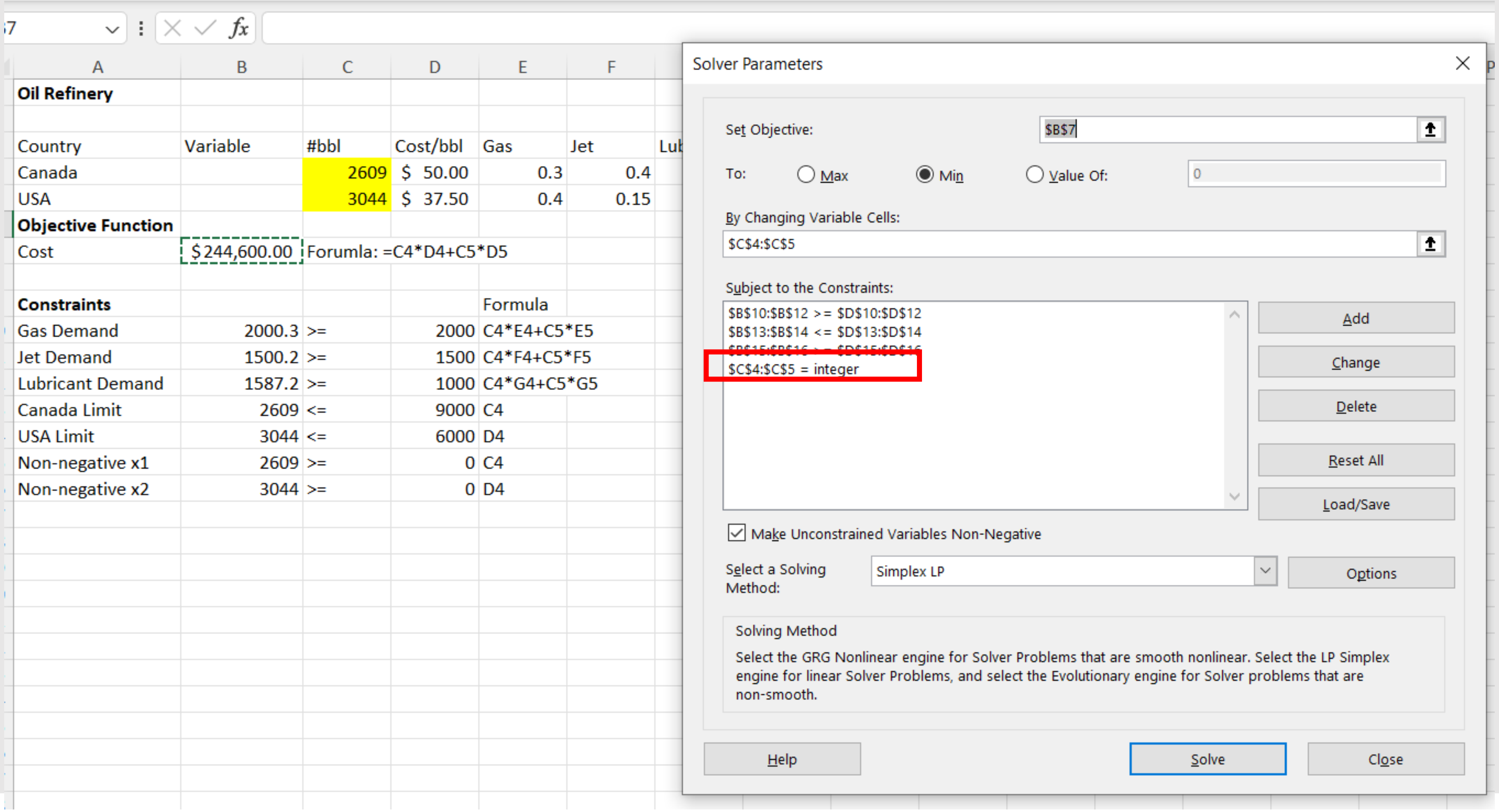
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$4	Canada #bbl	2608.695652	0	50	50	21.875
$C$5	USA #bbl	3043.478261	0	37.5	29.16666667	18.75

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$10	Gas Demand Variable	2000	65.2173913	2000	850	613.6363636
$B$11	Jet Demand Variable	1500	76.08695652	1500	1166.666667	750
$B$12	Lubricant Demand Variable	1586.956522	0	1000	586.9565217	1E+30
$B$13	Canada Limit Variable	2608.695652	0	9000	1E+30	6391.304348
$B$14	USA Limit Variable	3043.478261	0	6000	1E+30	2956.521739
$B$15	Non-negative x1 Variable	2608.695652	0	0	2608.695652	1E+30
$B$16	Non-negative x2 Variable	3043.478261	0	0	3043.478261	1E+30

LP with Integer Variables

→You can only purchase integer number of barrels.



A	B	C	D	E	F
Oil Refinery					
Country	Variable	#bbl	Cost/bbl	Gas	Jet
Canada		2609	$ 50.00	0.3	0.4
USA		3044	$ 37.50	0.4	0.15
Objective Function					
Cost	$244,600.00	Forumla: =C4*D4+C5*D5			
Constraints				Formula	
Gas Demand	2000.3	>=	2000	C4*E4+C5*E5	
Jet Demand	1500.2	>=	1500	C4*F4+C5*F5	
Lubricant Demand	1587.2	>=	1000	C4*G4+C5*G5	
Canada Limit	2609	<=	9000	C4	
USA Limit	3044	<=	6000	D4	
Non-negative x1	2609	>=	0	C4	
Non-negative x2	3044	>=	0	D4	

Add integer constraints to the decision variables

LP with Integer Variables

→Only answer report is available in Excel for integer problems (i.e. no sensitivity report)

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$B$7	Cost Variable	$ 244,565.22	$ 244,600.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$4	Canada #bbl	2608.695652	2609	Integer
$C$5	USA #bbl	3043.478261	3044	Integer

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$10	Gas Demand Variable	2000.3	$B$10>=$D$10	Not Binding	0.3
$B$11	Jet Demand Variable	1500.2	$B$11>=$D$11	Not Binding	0.2
$B$12	Lubricant Demand Variable	1587.2	$B$12>=$D$12	Not Binding	587.2
$B$13	Canada Limit Variable	2609	$B$13<=$D$13	Not Binding	6391
$B$14	USA Limit Variable	3044	$B$14<=$D$14	Not Binding	2956
$B$15	Non-negative x1 Variable	2609	$B$15>=$D$15	Not Binding	2609
$B$16	Non-negative x2 Variable	3044	$B$16>=$D$16	Not Binding	3044
$C$4:$C$5=Integer					

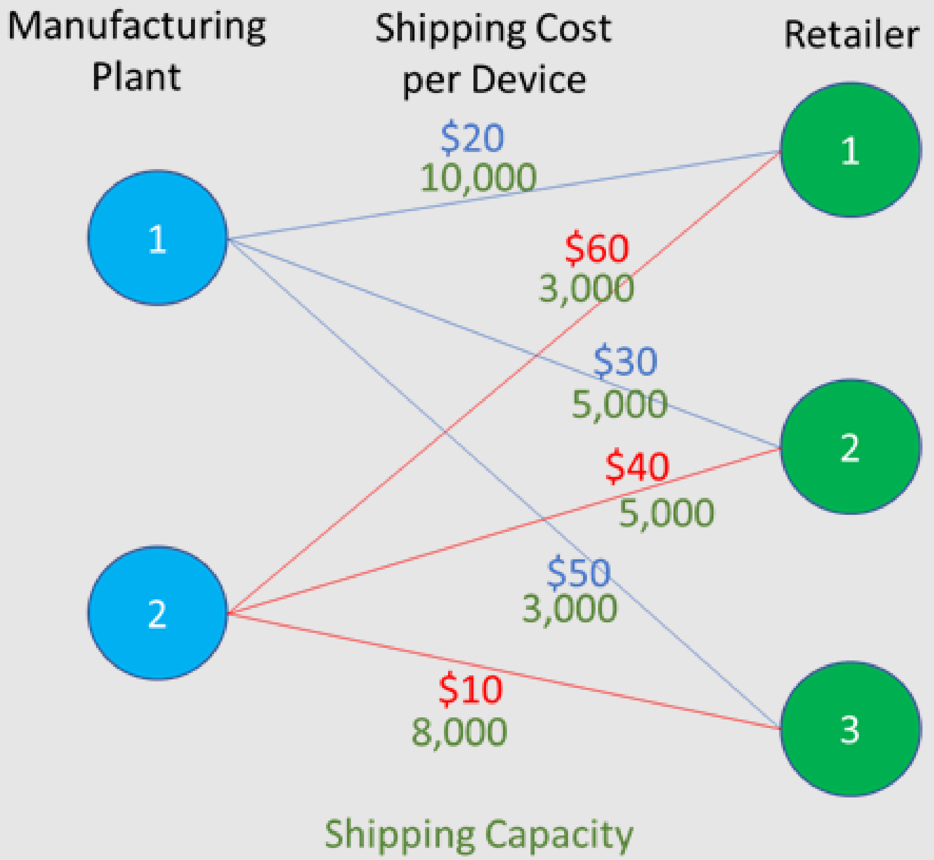
Linear Programming in Excel Example 2

Manufacturing Plant 1

Capacity	Cost/device
Device A: 10,000	$200
Device B: 5,000	$350
Device C: 7,000	$300
Total capacity: 15,000	

Manufacturing Plant 2

Capacity	Cost/device
Device A: 12,000	$150
Device B: 8,000	$300
Device C: 5,000	$350
Total capacity: 20,000	



Retailer 1

Demand	Price/device
Device A: 3,000	$500
Device B: 5,000	$650
Device C: 7,000	$600

Retailer 2

Demand	Price/device
Device A: 4,000	$450
Device B: 3,000	$650
Device C: 3,000	$650

Retailer 3

Demand	Price/device
Device A: 2,000	$400
Device B: 5,000	$600
Device C: 3,000	$550

Linear Programming in Excel Example 2

Maximize profit:

sale price –

(manufacturing cost + shipping cost)

$\rightarrow \max_{\boldsymbol{x}} \phi = \boldsymbol{c}^T \boldsymbol{X}$

→ s.t. →

Non-negative Constraints →

$$X_{i,j,k} \geq 0 \; \forall \, i,j,k$$

Demand Constraints

$$\sum_{j=1}^{2} X_{A,j,1} \leq 3{,}000$$
$$\sum_{j=1}^{2} X_{A,j,2} \leq 4{,}000$$
$$\sum_{j=1}^{2} X_{A,j,3} \leq 2{,}000$$
$$\sum_{j=1}^{2} X_{B,j,1} \leq 5{,}000$$
$$\sum_{j=1}^{2} X_{B,j,2} \leq 3{,}000$$
$$\sum_{j=1}^{2} X_{B,j,3} \leq 5{,}000$$
$$\sum_{j=1}^{2} X_{C,j,1} \leq 7{,}000$$
$$\sum_{j=1}^{2} X_{C,j,2} \leq 3{,}000$$
$$\sum_{j=1}^{2} X_{C,j,3} \leq 3{,}000$$

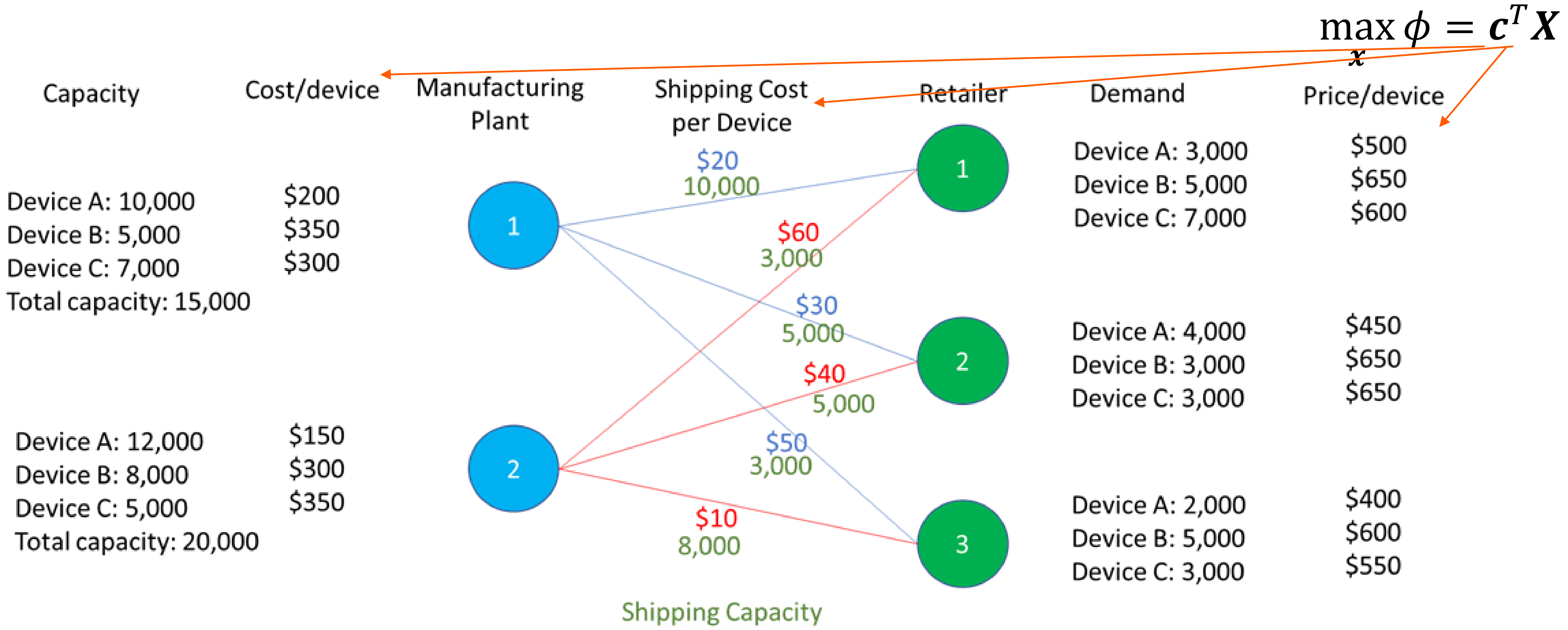
Manufacturing Capacity Constraints

$$\sum_{k=1}^{3} X_{A,1,k} \leq 10{,}000$$
$$\sum_{k=1}^{3} X_{A,2,k} \leq 12{,}000$$
$$\sum_{k=1}^{3} X_{B,1,k} \leq 5{,}000$$
$$\sum_{k=1}^{3} X_{B,2,k} \leq 8{,}000$$
$$\sum_{k=1}^{3} X_{C,1,k} \leq 7{,}000$$
$$\sum_{k=1}^{3} X_{C,2,k} \leq 5{,}000$$
$$\sum_{i=A}^{C} \sum_{k=1}^{3} X_{i,1,k} \leq 15{,}000$$
$$\sum_{i=A}^{C} \sum_{k=1}^{3} X_{i,2,k} \leq 20{,}000$$

Shipping Capacity Constraints

$$\sum_{i=A}^{C} X_{i,1,1} \leq 10{,}000$$
$$\sum_{i=A}^{C} X_{i,1,2} \leq 5{,}000$$
$$\sum_{i=A}^{C} X_{i,1,3} \leq 3{,}000$$
$$\sum_{i=A}^{C} X_{i,2,1} \leq 3{,}000$$
$$\sum_{i=A}^{C} X_{i,2,2} \leq 5{,}000$$
$$\sum_{i=A}^{C} X_{i,2,3} \leq 8{,}000$$

Linear Programming in Excel Example 2



Linear Programming in Excel Example 2

$$\max_{x} \phi = c^T X$$

Capacity	Cost/device	Manufacturing Plant
Device A: 10,000 Device B: 5,000 Device C: 7,000 Total capacity: 15,000	\$200 \$350 \$300	1
Device A: 12,000 Device B: 8,000 Device C: 5,000 Total capacity: 20,000	\$150 \$300 \$350	2

Demand Constraints

$$\sum_{j=1}^{2} X_{A,j,1} \leq 3{,}000$$
$$\sum_{j=1}^{2} X_{A,j,2} \leq 4{,}000$$
$$\sum_{j=1}^{2} X_{A,j,3} \leq 2{,}000$$
$$\sum_{j=1}^{2} X_{B,j,1} \leq 5{,}000$$
$$\sum_{j=1}^{2} X_{B,j,2} \leq 3{,}000$$
$$\sum_{j=1}^{2} X_{B,j,3} \leq 5{,}000$$
$$\sum_{j=1}^{2} X_{C,j,1} \leq 7{,}000$$
$$\sum_{j=1}^{2} X_{C,j,2} \leq 3{,}000$$
$$\sum_{j=1}^{2} X_{C,j,3} \leq 3{,}000$$

Demand	Price/device
Device A: 3,000 Device B: 5,000 Device C: 7,000	\$500 \$650 \$600
Device A: 4,000 Device B: 3,000 Device C: 3,000	\$450 \$650 \$650
Device A: 2,000 Device B: 5,000 Device C: 3,000	\$400 \$600 \$550

Linear Programming in Excel Example 2

$$\max_{\boldsymbol{x}} \phi = \boldsymbol{c}^T \boldsymbol{X}$$

Capacity

Cost/d

Device A: 10,000
Device B: 5,000
Device C: 7,000
Total capacity: 15,000

$20
$35
$30

$$\sum_{k=1}^{3} X_{B,2,k} \leq 8{,}000$$
$$\sum_{k=1}^{3} X_{C,1,k} \leq 7{,}000$$
$$\sum_{k=1}^{3} X_{C,2,k} \leq 5{,}000$$
$$\sum_{i=A}^{C}\sum_{k=1}^{3} X_{i,1,k} \leq 15{,}000$$
$$\sum_{i=A}^{C}\sum_{k=1}^{3} X_{i,2,k} \leq 20{,}000$$

Manufacturing Capacity Constraints

Device A: 12,000
Device B: 8,000
Device C: 5,000
Total capacity: 20,000

Manufacturing Capacity Constraints

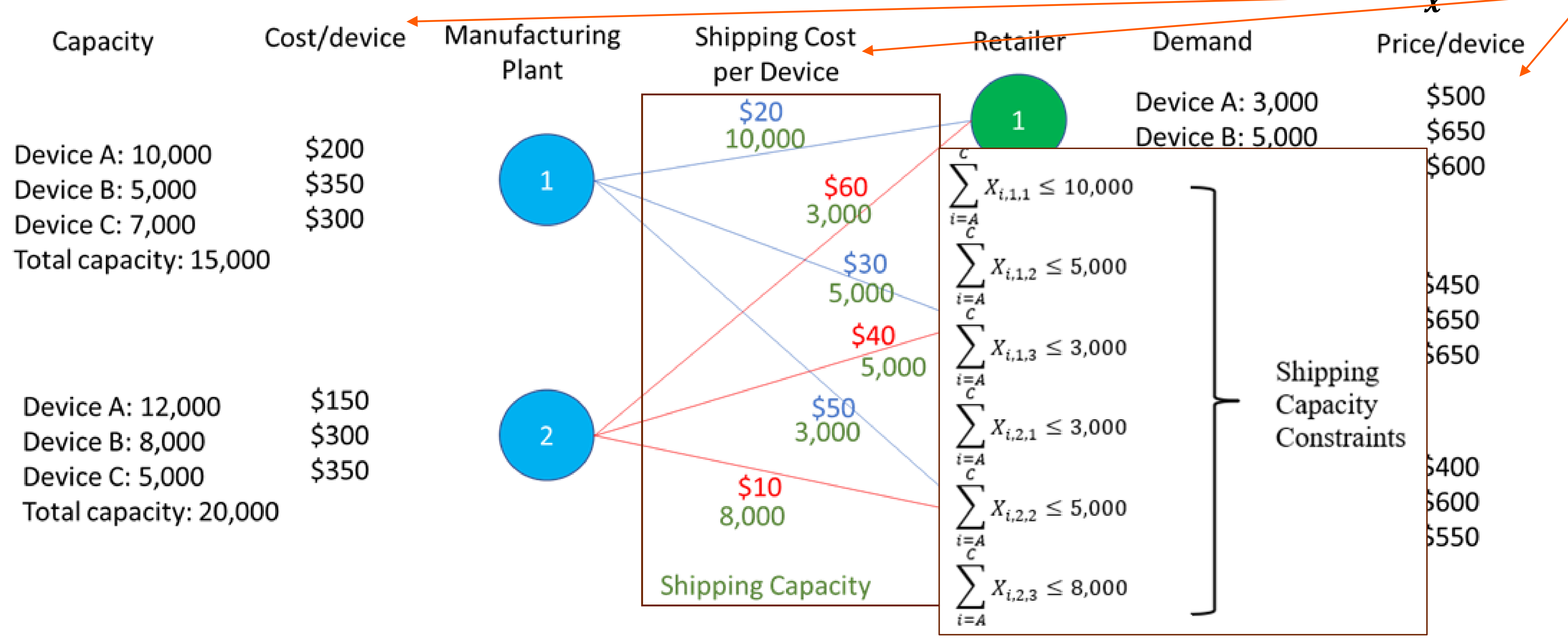
$$\sum_{k=1}^{3} X_{A,1,k} \leq 10{,}000$$
$$\sum_{k=1}^{3} X_{A,2,k} \leq 12{,}000$$
$$\sum_{k=1}^{3} X_{B,1,k} \leq 5{,}000$$

$40
5,000

Retailer	Demand	Price/device
1	Device A: 3,000 Device B: 5,000 Device C: 7,000	$500 $650 $600
2	Device A: 4,000 Device B: 3,000 Device C: 3,000	$450 $650 $650
3	Device A: 2,000 Device B: 5,000 Device C: 3,000	$400 $600 $550

Linear Programming in Excel Example 2

$$\max_{\boldsymbol{x}} \phi = \boldsymbol{c}^T \boldsymbol{X}$$



Formulating the Problem on Excel: Objective

O18

	A	B	C	D	E	F	G	H	I	J	K
1	Variable Index	Device	Plant	Retailer	Cost	Shipping	Price	Profit/device	Devices Made		Total Profit
2	A11	A	1	1	$ 200.00	$ 20.00	$ 500.00	$ 280.00	0		0
3	A12	A	1	2	$ 200.00	$ 30.00	$ 450.00	$ 220.00	0		
4	A13	A	1	3	$ 200.00	$ 50.00	$ 400.00	$ 150.00	0		
5	A21	A	2	1	$ 150.00	$ 60.00	$ 500.00	$ 290.00	0		
6	A22	A	2	2	$ 150.00	$ 40.00	$ 450.00	$ 260.00	0		
7	A23	A	2	3	$ 150.00	$ 10.00	$ 400.00	$ 240.00	0		
8	B11	B	1	1	$ 350.00	$ 20.00	$ 650.00	$ 280.00	0		
9	B12	B	1	2	$ 350.00	$ 30.00	$ 650.00	$ 270.00	0		
10	B13	B	1	3	$ 350.00	$ 50.00	$ 600.00	$ 200.00	0		
11	B21	B	2	1	$ 300.00	$ 60.00	$ 650.00	$ 290.00	0		
12	B22	B	2	2	$ 300.00	$ 40.00	$ 650.00	$ 310.00	0		
13	B23	B	2	3	$ 300.00	$ 10.00	$ 600.00	$ 290.00	0		
14	C11	C	1	1	$ 300.00	$ 20.00	$ 600.00	$ 280.00	0		
15	C12	C	1	2	$ 300.00	$ 30.00	$ 650.00	$ 320.00	0		
16	C13	C	1	3	$ 300.00	$ 50.00	$ 550.00	$ 200.00	0		
17	C21	C	2	1	$ 350.00	$ 60.00	$ 600.00	$ 190.00	0		
18	C22	C	2	2	$ 350.00	$ 40.00	$ 650.00	$ 260.00	0		
19	C23	C	2	3	$ 350.00	$ 10.00	$ 550.00	$ 190.00	0		
20											

Formulating the Problem on Excel: Constraints

	A	B	C	D	E
21	Constraints				
22	Name	Value		RHS	Formula
23	A Demand 1	0	<=	3,000	I2+I5
24	A Demand 2	0	<=	4000	I3+I6
25	A Demand 3	0	<=	2000	I4+I7
26	B Demand 1	0	<=	5000	I8+I11
27	B Demand 2	0	<=	3000	I9+I12
28	B Demand 3	0	<=	5000	I10+I13
29	C Demand 1	0	<=	7000	I14+I17
30	C Demand 2	0	<=	3000	I15+I18
31	C Demand 3	0	<=	3000	I16+I19
32	A Capacity 1	0	<=	10000	SUM(I2:I4)
33	A Capacity 2	0	<=	12000	SUM(I5:I7)
34	B Capacity 1	0	<=	5000	SUM(I8:I10)
35	B Capacity 2	0	<=	8000	SUM(I11:I13)
36	C Capacity 1	0	<=	7000	SUM(I14:I16)
37	C Capacity 2	0	<=	5000	SUM(I17:I19)
38	Total Capacity 1	0	<=	15000	SUM(I2:I4,I8:I10,I14:I16)
39	Total Capacity 2	0	<=	20000	SUM(I5:I7,I11:I13,I17:I19)
40	Shipping Capacit	0	<=	10000	I2+I8+I14
41	Shipping Capacit	0	<=	5000	I3+I9+I15
42	Shipping Capacit	0	<=	3000	I4+I10+I16
43	Shipping Capacit	0	<=	3000	I5+I11+I17
44	Shipping Capacit	0	<=	5000	I6+I12+I18
45	Shpping Capacity	0	<=	8000	I7+I13+I19

Solution Diagram