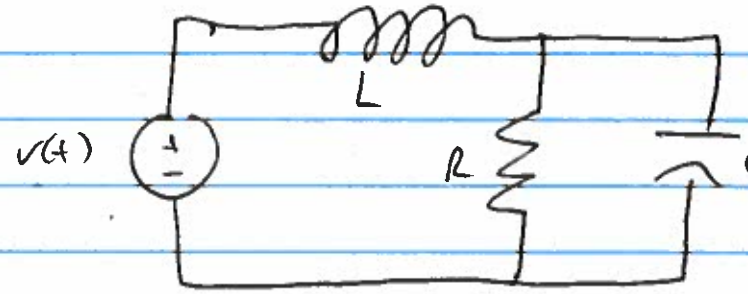
State Space Representation of Electrical Network



Input = $v(t)$

output = $i_R(t)$

$$V_L(t) = L\frac{di_L(t)}{dt} \Rightarrow \frac{di_L(t)}{dt} = \frac{1}{L}V_L(t)$$

$$V_c(t) = \frac{1}{C}\int_0^t i_c(u)\,du$$

$$\frac{dV_c(t)}{dt} = \frac{1}{C}i_c(t)$$

State variables: $V_c(t)$, $i_L(t)$

Input variable: $v(t)$

output variable: $i_R(t)$

→ want to write $di_L(t)/dt$ and $dv_c(t)/dt$ in terms of state variables.

$$\frac{di_L(t)}{dt} = \frac{1}{L}V_L(t) = \frac{1}{L}v(t) - \frac{1}{L}V_c(t) \quad \checkmark$$

$$i_L(t) = i_R(t) + i_c(t)$$

$$i_R(t) = \frac{V_c(t)}{R} \Rightarrow i_c(t) = i_L(t) - \frac{V_c(t)}{R}$$

$$\frac{dV_c(t)}{dt} = \frac{1}{C}\left(i_L(t) - \frac{V_c(t)}{R}\right)$$

So, $x(t) = \begin{bmatrix} V_c(t) \\ i_L(t) \end{bmatrix}$

$$A = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{L} \\ -\frac{1}{L} & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}$$

$$C = \begin{bmatrix} 1/R & 0 \end{bmatrix} \qquad D = 0$$