Lab 5, SFWR ENG 3DX4

Steady State Error and PID Control of Electromechanical Systems

PRELAB EXERCISES

Prelabs due the week of: April 1, 2024

1. In Lab 4, we used a PD compensator to control our ball and beam apparatus. The transfer function of our PD compensator was as follows:

$$G_c(s) = (K_D s + K_P)$$

However, we did not use the compensator in this form. The transfer function we used in lab was as follows:

$$G_c(s) = K_C(s + z)$$

Solve for K_C and z in terms of K_D and K_P

2. Given that the transfer function of our Ball and Beam plant used in the previous lab is:

$$G(s) = \frac{0.419}{s^2}$$

And given that the controller is applied to the plant in cascade configuration, find:

   (a) Static error constant for position (position constant).
   (b) Static error constant for velocity (velocity constant).
   (c) Static error constant for acceleration (acceleration constant).
   (d) Steady state error for a step input $u(t)$.
   (e) Steady state error for a ramp input $tu(t)$.
   (f) Steady state error for a parabolic input $t^2u(t)$.

3. In Lab 5, we will be augmenting our controller to include an integrator. The transfer function of our new PID compensator will be as follows:

$$G_c(s) = (K_D s + K_P + \frac{K_I}{s}) \tag{1}$$

Given that the transfer function for our plant has not changed, and given that this controller is also applied to the plant in cascade configuration, find:

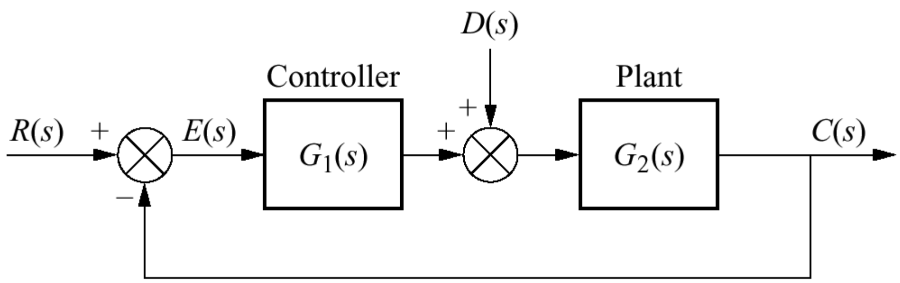
   (a) Static error constant for position (position constant).
   (b) Static error constant for velocity (velocity constant).
   (c) Static error constant for acceleration (acceleration constant).

(d) Steady state error for a step input $u(t)$.

(e) Steady state error for a ramp input $tu(t)$.

(f) Steady state error for a parabolic input $t^2u(t)$.

4. Consider the block diagram shown below where $D(s)$ is a step disturbance input.



Ideally you want your controller design to reject a step disturbance input at $D(s)$. This means that in the steady state for $D(s) = \frac{1}{s}$, the value of $Y(s)$ is unchanged.

(a) Ignoring the input $R(s)$, what is the transfer function $\frac{E(s)}{D(s)}$ in terms of $G_1(s)$ and $G_2(s)$?

(b) For $G_1(s) = K_C(s + z)$ and $G_2(s) = \frac{0.419}{s^2}$ what is the steady state error resulting from step inputs $R(s) = \frac{A}{s}$ and $D(s)\frac{B}{s}$.

(c) For $G_1(s) = (K_D s + K_P + \frac{K_I}{s})$ and $G_2(s) = \frac{0.419}{s^2}$ what is the steady state error resulting from step inputs $R(s) = \frac{A}{s}$ and $D(s)\frac{B}{s}$.

Hint: Its a linear system so you can consider the steady state error resulting from $R(s)$ and the steady state error resulting from $D(s)$ separately and then sum them!