SFWR ENG 3DX4 – Answers to Sample Questions for Test

1. The input has Laplace transform $\frac{1}{s+2}$. The output is thus

$$Y(s) = \frac{-0.5}{s+2} + \frac{0.1}{s+4} + \frac{0.4}{s-1}$$

   Do inverse Laplace transform to get $y(t)$.

2.

$$\begin{aligned} Y(s) &= \frac{1}{s}\frac{s^2+30s+360}{(s+10)(s+6)^2} \\ &= \frac{1}{s} - \frac{1}{s+10} - \frac{9}{(s+6)^2} \end{aligned}$$

   Do inverse Laplace transform, the only thing to note here is that the Laplace transform of te^{-at} is $\frac{1}{(s+a)^2}$.

3. Done in lecture.

4. Let $I_1(s)$ be the cw current in left loop, $I_2(s)$ be the cw current in the right loop. Then

$$\begin{aligned} V_i(s) - 2sI_1(s) - (I_1(s) - I_2(s)) &= 0 \\ -3sI_2(s) - \frac{1}{2s}I_2(s) - (I_2(s) - I_1(s)) &= 0 \end{aligned}$$

   Solve for $I_2(s)$ in terms of $V_i(s)$, then use $V_o(s) = 3sI_2(s)$ to get the transfer function.

5. Done in lecture.

6. Done in lecture.

7. (a) Controllable form is one possibility:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [6 \quad 0 \quad 0]x$$

   (b)

$$\begin{aligned} \dot{\bar{x}}_1 &= \dot{x}_1 + \dot{x}_2 \\ &= x_2 + x_3 \\ &= \bar{x}_3 - \bar{x}_1 + 2\bar{x}_2 \\ \dot{\bar{x}}_2 &= \dot{x}_2 \end{aligned}$$

$$\begin{aligned}
&= x_3 \\
&= \bar{x}_3 - \bar{x}_1 + \bar{x}_2 \\
\dot{\bar{x}}_3 &= \dot{x}_1 + \dot{x}_3 \\
&= x_2 - 6x_1 - 11x_2 - 6x_3 + u \\
&= -6\bar{x}_3 - 10\bar{x}_2 + u \\
y &= 6x_1 \\
&= 6\bar{x}_1 - 6\bar{x}_2
\end{aligned}$$

So,

$$\dot{\bar{x}} = \begin{bmatrix} -1 & 2 & -1 \\ -1 & 1 & 1 \\ 0 & -10 & -6 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [6 \quad -6 \quad 0]\bar{x}$$

8. Done in lecture. Note that you could also choose

$$\begin{aligned}
x_1 &= \theta \\
x_2 &= \frac{d\theta}{dt} \\
x_3 &= i
\end{aligned}$$

and get a valid three-dimensional system. You could check that the two-dimensional representation from lecture and that from the the three-dimensional system are the same (as they should be).

9. Time to go to 63% of the final value is roughly 5.5 seconds. Final value is 10, so transfer function is

$$\frac{10/5.5}{s + 1/5.5}$$

10. Closed-loop transfer function is

$$\frac{N(s)}{s^3 + (2 + K_p)s^2 + (1 + 5K_p)s + 25K_p}$$

The Routh table is

s^3	1	$1 + 5K_p$
s^2	$2 + K_p$	$25K_p$
s^1	$\frac{(1+5K_p)(2+K_p)-25K_p}{2+K_p}$	0
s^0	$25K_p$	0

From the last row we need $K_p > 0$. From the row above that, we need $5K_p^2 - 14K_p + 2 > 0$, or $(K_p - 2.65)(K_p - 0.15) > 0$, from which we conclude that we need $K_p > 2.65$ or $0 < K_p < 0.15$.

11. Closed loop transfer function is

$$\frac{K}{s^2 + 5s + K}$$

If we fix the overshoot requirement, we get $\zeta = 0.591$. Now, with $2\zeta\omega_n = 5$ we have $\omega_n = 4.230$, which gives a T_p of 0.9207. So, as long as the requirement is that the peak time is less than 1 second, both requirements can be met.

12. Done in lecture.

13. The open loop is $\frac{K}{10s+1}$. Now, $K_p = \lim_{s\to 0} G(s) = K$, so steady-state error for a step function is $1/(1+K)$. Thus, need $K \geq 9$. If $K = 9$, closed loop transfer function is

$$\frac{K}{10s + K + 1}$$

When $K = 9$, the system is stable.

14. Note that there were issues in the question, but hopefully you figured out what was meant (the second $\dot{x}_2(t)$ should have been $\dot{x}_3(t)$ and the $x(t)$ was missing after the A matrix). In any case, the eigenvalues of A are at -3, $-\frac{3}{2} \pm \frac{\sqrt{(1-4a)}}{2}$. For all poles to be in the LHP, need that the real part of $-\frac{3}{2} + \frac{\sqrt{(1-4a)}}{2}$ is less than zero, which occurs when $a > -2$.

15. (a) Closed loop is

$$\frac{K_1 s + K_2}{s^2 + (10 + K_1 s) + K_2}$$

Easy to check that $K_1 > -10$ and $K_2 > 0$ is needed for stability.

(b) $K_v = \lim_{s\to 0} sG(s) = 1/10$, so steady-state error is 10.

(c) Could change controller to $\frac{K_1 s + K_2}{s^2}$, but would need to check stability so that FVT can be used.