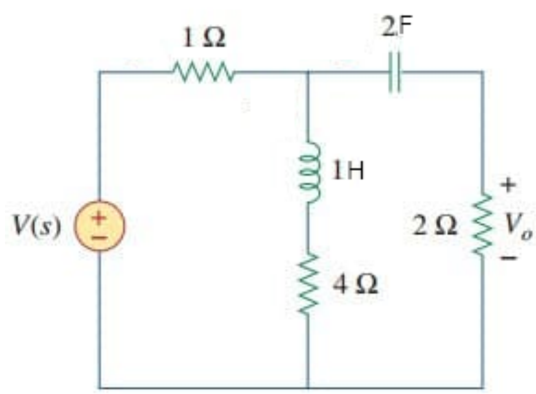
SFWR ENG 3DX4 – Sample Questions for Test

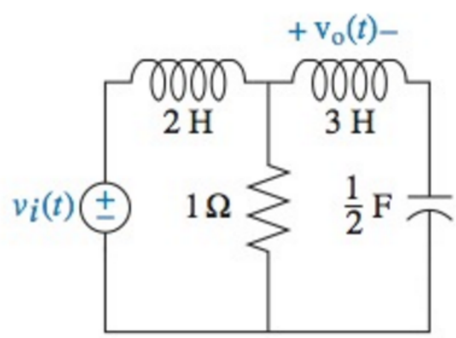
1. Assume that you are given a system with $G(s) = \frac{s+5}{s^2+3s-4}$. What is the output (in the time domain) if the input is $e^{-2t}u(t)$?

2. Give the step response (in the time domain) of the system $G(s) = \frac{s^2+30s+360}{(s+10)(s+6)^2}$.

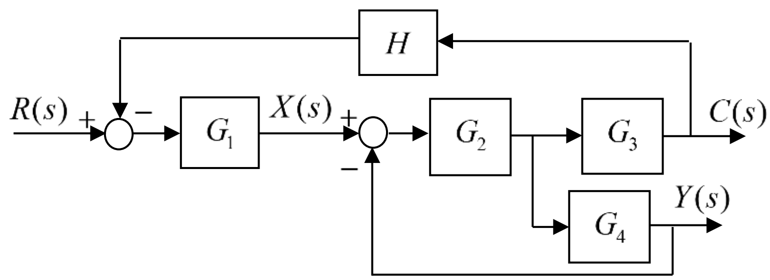
3. Find the transfer function $V_o(s)/V(s)$ for the following circuit:



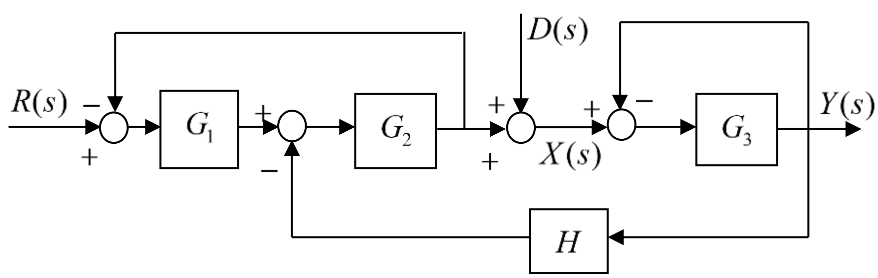
4. Find the transfer function $V_o(s)/V_i(s)$ for the following circuit:



5. Find the transfer function $Y(s)/R(s)$ for the following block diagram:



6. Find the transfer function $Y(s)/D(s)$ for the following block diagram:



7. A system has transfer function

$$\frac{6}{s^3 + 6s^2 + 11s + 6}$$

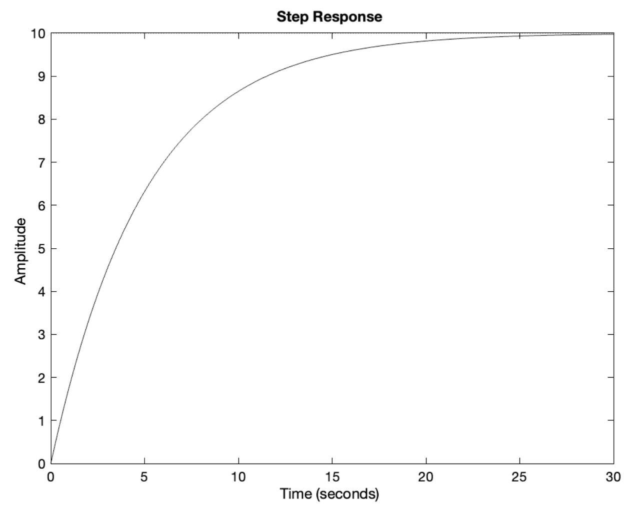
   (a) Give a state space representation of this system.
   (b) You should have had three state variables in the previous part, call them $x_1(t)$, $x_2(t)$ and $x_3(t)$. Suppose that we define $\bar{x}_1(t) = x_1(t) + x_2(t)$, $\bar{x}_2(t) = x_2(t)$ and $\bar{x}_3(t) = x_1(t) + x_3(t)$. What is the state space representation for this set of state variables?

8. Suppose we have a system described by the following set of differential equations:

$$\begin{aligned} \frac{d^2}{dt^2}\theta(t) + 10\frac{d}{dt}\theta(t) &= i(t) \\ \frac{d}{dt}i(t) + 2i(t) &= V(t) - 0.02\frac{d}{dt}\theta(t) \end{aligned}$$

   If $V(t)$ is the input and $\frac{d}{dt}\theta(t)$ is the output, give a state-space representation of this system. Is the system stable?

9. Estimate the transfer function for a system with the following step response:



10. A system is in the unity-gain configuration: controller and plant in series, with negative unity feedback from the output back to the input. The plant's transfer function is

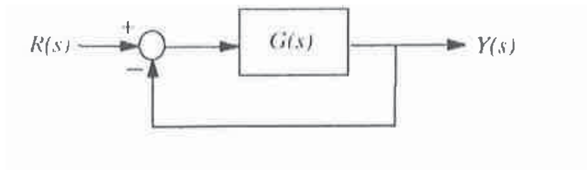
$$G(s) = \frac{5}{(s+1)^2}$$

and the controller is

$$G_{PID}(s) = K_p\left(1 + \frac{5}{s} + 0.2s\right)$$

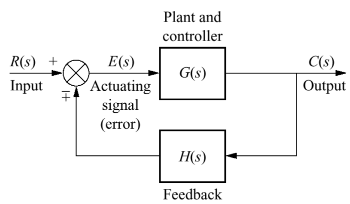
For what range of K_p is the closed-loop system stable?

11. Consider the following unity feedback system:



Suppose that $G(s) = \frac{K}{s(s+5)}$. Is it possible to meet the following requirements: $T_p \leq 1.0$ seconds and $\%OS \leq 10\%$? If they both cannot be met, can one of them be met? Justify your answer.

12. Consider a system with feedback form:

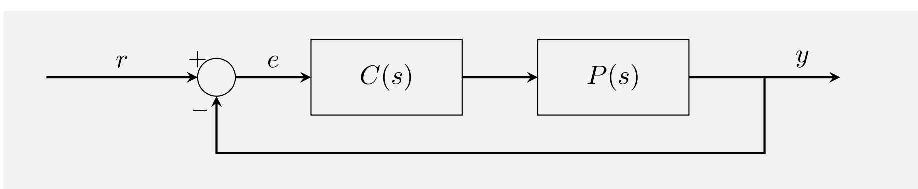


It is negative feedback, so the signal fed through $H(s)$ is subtracted from $R(s)$. Suppose that

$$\begin{aligned} G(s) &= \frac{1}{s^2 + 2s + 16} \\ H(s) &= Ts + K \end{aligned}$$

Choose T and K such that the natural frequency is 3 rad/s and the damping ratio is 0.8.

13. Consider the system described by the following block diagram:



Suppose that $C(s) = K$ and $P(s) = \frac{1}{10s+1}$. Find the smallest value of K such that the closed-loop system is stable and when $r(t)$ is a unit step function, $\lim_{t\to\infty} |e(t)| \leq 0.1$.

14. Consider the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -a & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

For what values of a is the system stable?

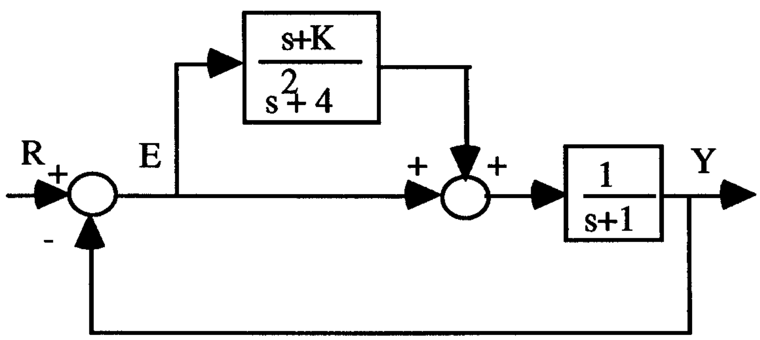
15. Consider the system from question 13, but now let $C(s) = \frac{K_1 s + K_2}{s}$ and $P(s) = \frac{1}{s+10}$.

    (a) Find the range of K_1 and K_2 such that the closed-loop system is stable.

    (b) Suppose that $K_1 = K_2 = 1$, what is the steady-state error for a unit ramp input ($r(t) = tu(t)$)?

    (c) If your steady-state error in (b) is not zero, how would you modify $C(s)$ to ensure that this error is zero? Justify your answer.

16. Consider the following system:



    (a) Suppose that the reference input is a sinusoid with frequency of 2 rad/s. What is the steady-state error?

    (b) For what range of K is the system stable?