Introduction

- Root locus graphically displays both transient response and stability information.

- The root locus allows us to choose the needed gain to get the desired transient response.

- Problem: when we choose different gains, we can only achieve responses that correspond to roots that are actually on the root locus.
- By adding a controller in series with the original system, we can add new poles to the system, located at the desired spot.

Improving Transient Response

- If point B in Fig. 9.1 represented our desired transient response, represented by desired percent overshoot and settling time, we are out of luck.

- Using gain response, we can achieve the desired percent overshoot, but *not* the settling time at point A.

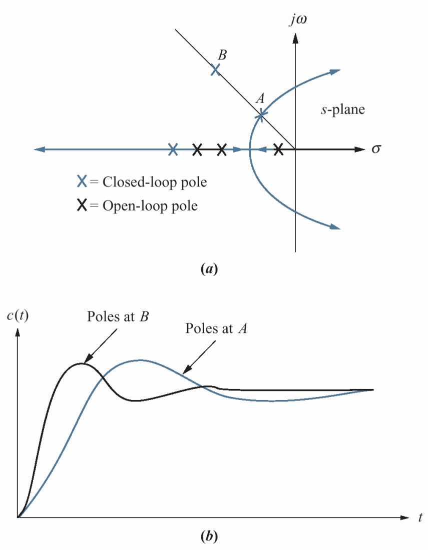


Figure 9.1

Improving Transient Response II

- Replacing original system would be expensive and may be difficult to find system with correct response and still satisfy other needed properties such as speed, power, durability etc.
- Instead we augment, or compensate, system by adding additional poles and zeros to get desired behavior.
- Disadvantage: Method increases order of system, which can also affect the response.
- Our design process will determine the location of the second-order closed-loop poles but not the higher-order ones.
- Must always evaluate response via simulation at end to ensure that actual higher-order system behaves as desired.

Improving Steady State Error (SSE)

- Can also add compensators to improve steady state error.
- Typically, increasing gain produces smaller steady state error, but larger overshoot.
- When we optimize gain for transient response, we can make steady state error worse.
- We can improve steady state error separate from gain by adding a compensator with an integrator ($\frac{1}{s}$) in feed forward path.

Compensator Terminology

- Proportional control systems are compensators that feed the error signal directly to the plant.
- Integral control systems feed the integral of the error to the plant.
- Derivative control systems feed the derivative of the error to the plant.

Configurations

- Two main configurations are cascade compensators and feedback compensators.
- Both methods can be used to change the open-loop poles and zeros location, and thus the root locus.

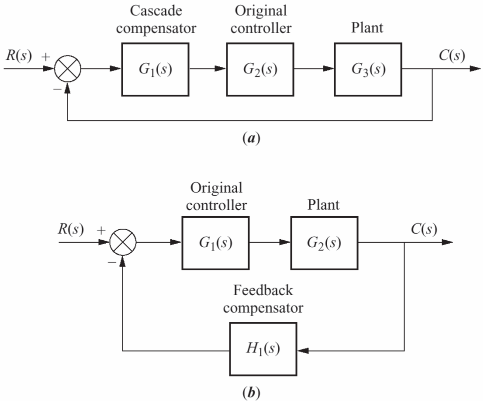


Figure 9.2.

Improving SSE via Cascade Compensation

- Two approaches: ideal integral compensation and lag compensation.
- First method will produce zero steady state error, but requires active circuit such as an op amp circuit (if implemented in analog).
- Second method will only reduce the error, but can be realized using passive circuits (if implemented in analog).

Note: If the compensator is done in software there is only an increase in complexity of the software, not the system hardware!

Ideal Integral Compensation

- Method involves adding an integrator ($\frac{1}{s}$) in feed forward path.
- Increases system type, causing error to go to zero.
- As implementation of method consists of both feeding the error and its integral to the plant, we use term proportional-plus-integral (PI) controller.

Ideal Integral Compensation: II

- Simply adding a pole at the origin would change the transient response of the original system.

- We see in example below, the desired pole location at A is now no longer on the root locus.

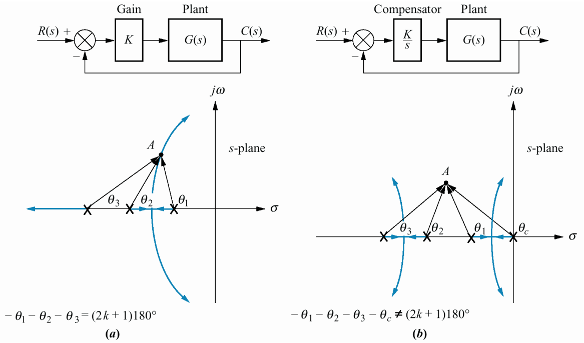
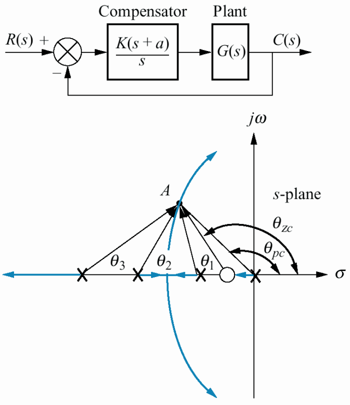


Figure 9.3.

Ideal Integral Compensation: Add Zero too!

- Fix this by adding a zero near the origin at $s = -a$.
- The angular contribution of the two effectively cancel.
- Require gain (K) is about the same as the ratio of the pole and zero magnitudes is about unity.

Figure 9.3



$$-\theta_1 - \theta_2 - \theta_3 - \theta_{pc} + \theta_{zc} \cong (2k+1)180°$$

(c)

Ideal Integral Compensation: Implementation

- Can implement controller as

$$G_c(s) = K_1 + \frac{K_2}{s} = \frac{K_1 s}{s} + \frac{K_2}{s} = \frac{K_1(s + \frac{K_2}{K_1})}{s}$$

- Example of a proportional-plus-integral controller.

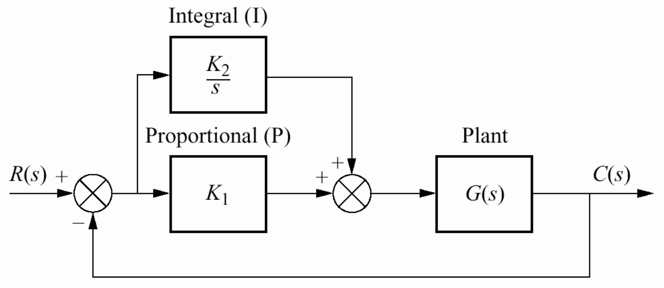


Figure 9.8.

Effect of Ideal Integral Compensation

- In example below, can we add the compensator shown to reduce SSE to zero but have small effect on the transient response?

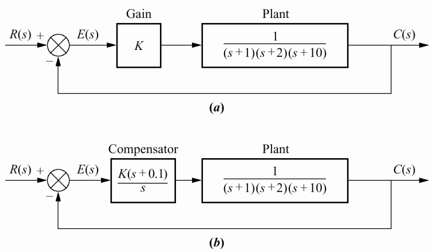


Figure 9.4.

Effect of Ideal Integral Compensation II

- Figure shows uncompensated system and desired dominant poles.

- Specification was for damping ratio of 0.174, thus angle $100.02^\circ = 180^o - \cos^{-1}(0.174)$.
- System has

$$
\begin{aligned}
K_p &= \lim_{s \to 0} KG(s) \\
&= \frac{164.6 \times 1}{1 \times 2 \times 10} = 8.23
\end{aligned}
$$

which gives $e_{ss} = \frac{1}{1+K_p} = 0.108$.

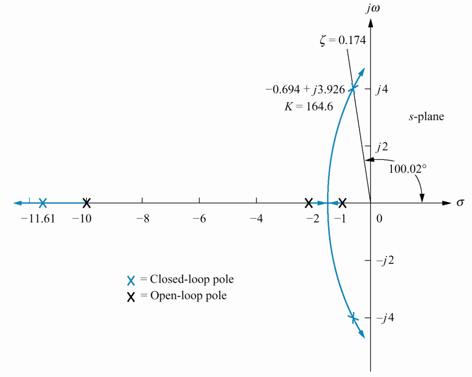


Figure 9.5

Effect of Ideal Integral Compensation III

- Figure shows system with added compensator.
- System responds to step input with zero error. Has gain and original three poles very close to previous values.
- Fourth closed loop pole at $s = -0.0902$ so essentially cancels with zero at $s = -0.1$.

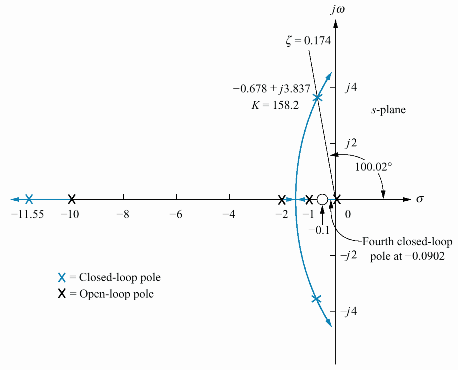


Figure 9.6

Effect of Compensation: Output Simulation

- Figure shows step response of original and compensated system.
- Original system takes 6 seconds to reach $\pm 2\%$ of final value while compensated system takes 18 seconds.
- Compensated system reaches $\pm 2\%$ of original final value in about same time. Extra time is to reach new final value.

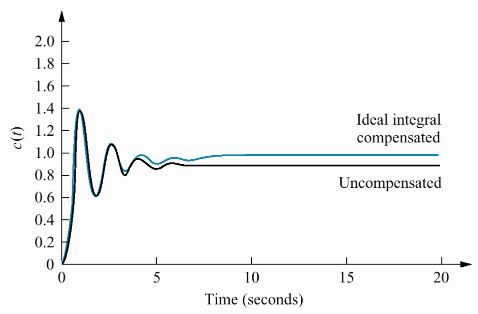


Figure 9.7.

Lag Compensation

- Require active circuit to put pole at origin.
- Passive circuits can only put a pole near origin, but can still improve steady state error.

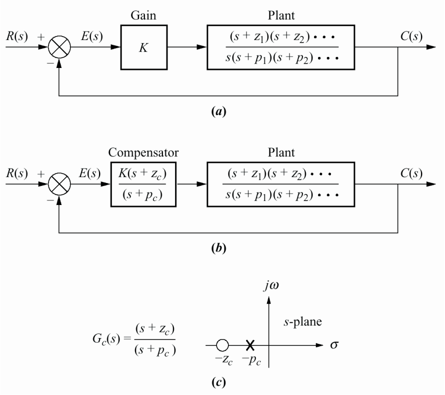


Figure 9.9

Lag Compensation II

- For type one system shown, $e_{ss} = \frac{1}{K_v}$.
- For original system we have $K_{v_O} = \frac{K z_1 z_2 \cdots}{p_1 p_2 \cdots}$.
- For compensated system, we have $K_{v_N} = \frac{(K z_1 z_2 \cdots)(z_c)}{(p_1 p_2 \cdots)(p_c)} = K_{v_O} \times \frac{z_c}{p_c}$.
- Increasing K_v will decrease SSE.

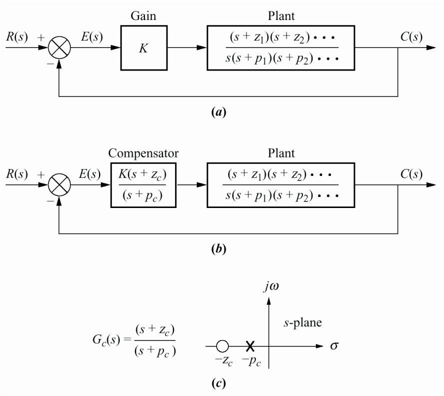


Figure 9.9

Lag Compensation III

- New steady state error is $e_{ss_O} \times \frac{p_c}{z_c}$.
- Need to maximize $\frac{z_c}{p_c}$ ratio to reduce SSE, but need the zero and pole to be close together so their effect cancels.
- Means z_c and p_c must both be small, thus close to the origin.

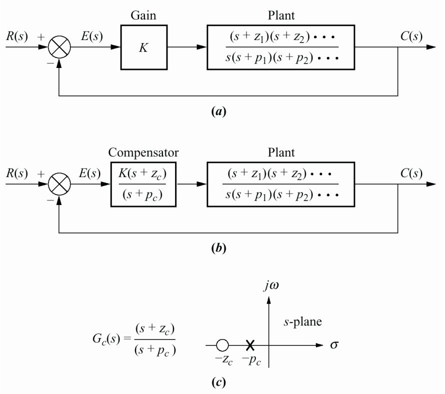


Figure 9.9

Improving Transient Response via Compensation

- Goal: design system with desired percent overshoot, but better settling time than uncompensated system.
- Two approaches: ideal derivative compensation and lead compensation.
- Involves adding a zero (differentiator) to the forward path.
- Differentiation is noisy. Can add a large unwanted signal.

Ideal Derivative Compensation

- Adding only a zero to forward path requires active circuit.
- As implementation of method consists of both feeding the error and its derivative to the plant, we use term proportional-plus-derivative (PD) controller.
- Goal is to reshape root locus so that desired closed loop pole is on root locus.

Ideal Derivative Compensation II

- To add a zero to forward path, we add compensator of form:

$$G_c(s) = s + z_c$$

- Choosing the right location for the zero can quicken the response of the original system.

- For step input and position control, system responds to sudden large change in input.

- This means that the derivative of this quick change will produce an even larger signal, driving the system even faster forward.

Ideal Derivative Compensation Design

- We first identify location of closed-loop poles with desired overshoot and settling time.
- The difference between 180^o and $\angle KG(s)H(s)$ to this pole location is the needed angular contribution of the compensator zero.
- For system below, design PD controller to yield 16% overshoot, and a threefold reduction in settling time.

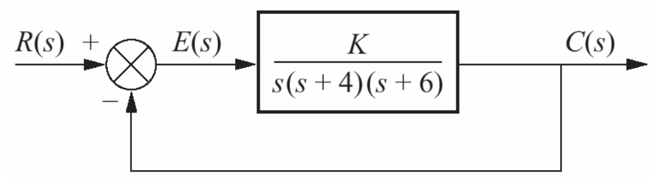


Figure 9.17.

Ideal Derivative Compensation Design II

- 16% overshoot corresponds to $\zeta = 0.504$, thus angle $120.26^o = 180^o - \cos^{-1}(0.504)$.
- Searching along this line of damping ratio discovers closed loop poles at $-1.205 \pm j2.064$ with gain $K = 43.35$.
- Settling time of system is $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{1.205} = 3.320$.

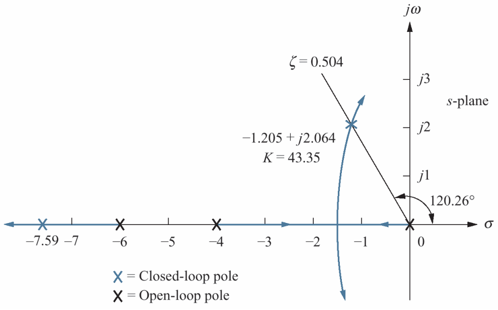


Figure 9.18.

Ideal Derivative Compensation Design III

- Gain of $K = 43.35$ gives us a third closed loop pole at $s = -7.59$ which is more than six times farther from the $j\omega$ axis than dominant poles.
- Our 2nd order approximation is thus valid for overshoot and settling time.

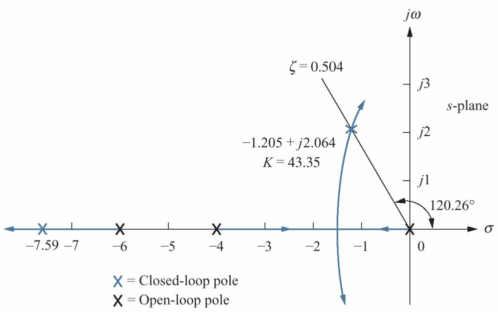


Figure 9.18.

Ideal Derivative Compensation Design IV

- Desired settling time is $\frac{3.320}{3} = 1.107$.
- Real part of desired pole is $\sigma = \frac{4}{T_s} = \frac{4}{1.107} = 3.613$.
- Imaginary part is $\omega_d = 3.613 \tan(180^o - 120.26^o) = 6.193$.

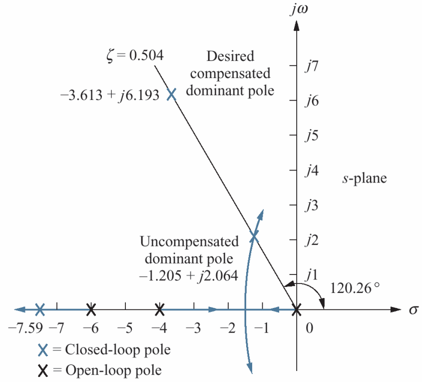


Figure 9.19.

Ideal Derivative Compensation Design V

- For a point to be on locus, must satisfy $\angle KG(s)H(s) = (2k+1)180^o$.
- For $s = -3.613 + j6.193$, we get sum of angles to be -275.6^o.

- Zero must contribute $\theta = 275.6^o - 180^o = 95.6^o$.
- Location of zero must satisfy $\tan(180^o - 95.6^o) = \frac{\text{opp}}{\text{adj}} = \frac{6.193}{3.613-\sigma}$.
- We thus need a zero at $s = -3.006$.

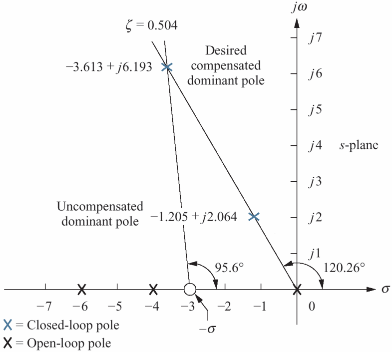


Figure 9.20

Ideal Derivative Compensation Design VI

- To get desired dominant poles, we need a gain of $K = 47.54$.
- At this gain, we get a third pole at $s = -2.755$.
- Second order approximation may not be valid as not close enough to get good pole-zero cancellation.

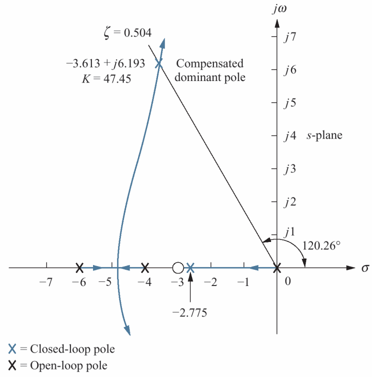


Figure 9.21.

Ideal Derivative Compensation Design VII

	Uncompensated	Simulation	Compensated	Simulation
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$		$\frac{K(s+3.006)}{s(s+4)(s+6)}$	
Dominant poles	$-1.205 \pm j2.064$		$-3.613 \pm j6.193$	
K	43.35		47.45	
ζ	0.504		0.504	
ω_n	2.39		7.17	
$\%OS$	16	14.8	16	11.8
T_s	3.320	3.6	1.107	1.2
T_p	1.522	1.7	0.507	0.5
K_v	1.806		5.94	
$e(\infty)$	0.554		0.168	
Third pole	-7.591		-2.775	
Zero	None		-3.006	
Comments	Second-order approx. OK		Pole-zero not canceling	

Table 9.3.

Ideal Derivative Compensation Design VIII

- Figure shows step response of uncompensated and compensated system.

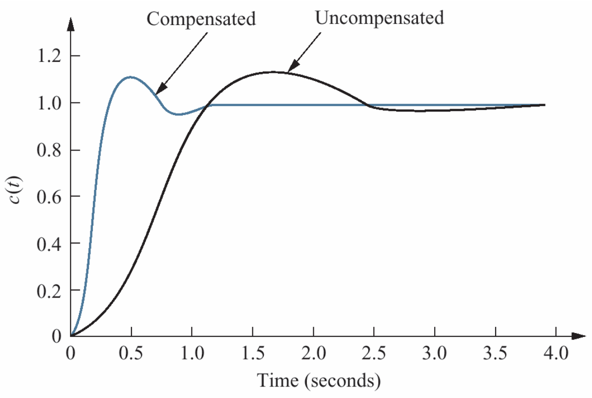


Figure 9.22.

PD Controller

- We implement an ideal derivative controller as a PD controller shown below.
- It has transfer function

$$G_c(s) = K_2 s + K_1 = K_2(s + \frac{K_1}{K_2})$$

- Choose K_2 to contribute to the required gain, and choose $\frac{K_1}{K_2}$ to position the zero at the correct place.

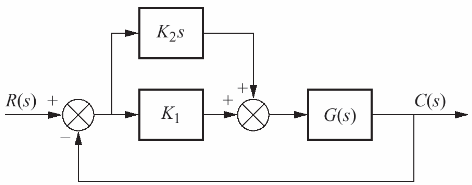


Figure 9.23.

Lead Compensation

- Advantages of a passive lead compensator
  - doesn't require additional power supply
  - reduced noise due to differentiation
- Approximates ideal derivative compensator by adding a zero and a pole.
- As long as pole farther to the left than zero, net angular contribution is still positive.

Lead Compensation II

- As for a PD controller, we first determine the location of the closed loop dominant poles with desired properties.
- The angular contribution of zero, z_c, is θ_2 and contribution of pole, p_c, is θ_1.
- Net contribution of pole-zero pair is thus $\theta_c = \theta_2 - \theta_1$.
- Choose θ_1 and θ_2 such that $\theta_2 - \theta_1 - \theta_3 - \theta_4 + \theta_5 = (2k+1)180^o$

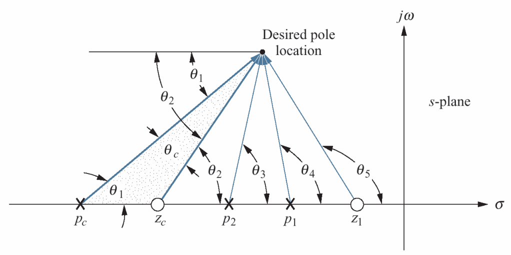


Figure 9.24.

Lead Compensation III

- Have infinite possible pole-zero combinations that would work.
- Compensators differ in such things as static error constants, required gain, quality of 2nd order approximation, and the shape of actual transient response.

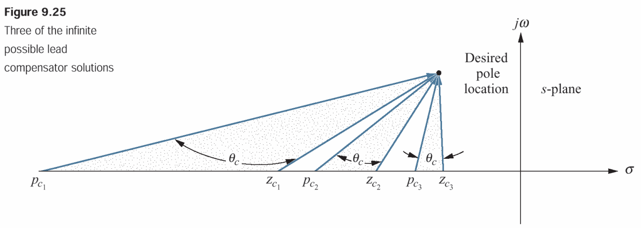


Figure 9.25.

Improving SSE and Transient Response

- To improve both transient response and steady state error, we can:
  - First design an active PD controller, and then an active PI controller. Called a proportional-plus-integral-plus-derivative (PID) controller.
  - First design a passive lead compensator, then design a passive lag compensator. Called a lead-lag compensator.

PID Controller

- Transfer function of controller is

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_3(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3})}{s}$$

- System has two zeros plus a pole at origin.

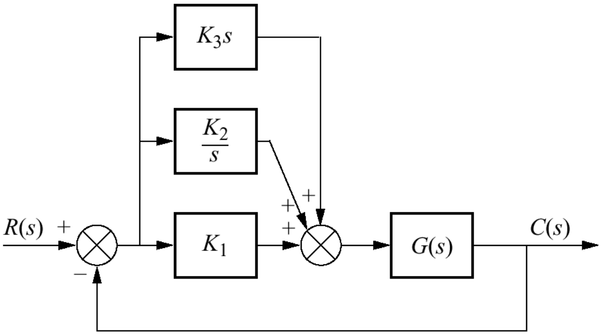


Figure 9.30.

PID Controller Design

1. Evaluate performance of original system.
2. Design PD controller for transient response improvement.
3. Simulate system to be sure requirements met.
4. Redesign if simulation shows they are not.
5. Design PI controller for steady state error requirement.
6. Determine gains K_1, K_2, K_3.
7. Simulate system to be sure all conditions met.
8. Redesign if not met.

PID Controller Design Example

1. Design PID controller so system has 20% overshoot, zero steady state error for step input, and peak time that is $\frac{2}{3}$ that of original system.

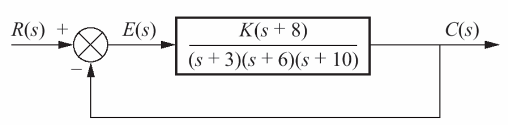


Figure 9.31.

PID Controller Design Example II

- 20% overshoot gives us $\zeta = 0.456$ at a line at 117.13^o.
- Gives dominant poles at $s = -5.415 \pm j10.57$, third pole at $s = -8.169$, and estimated peak time of 0.297 seconds.

- Desired pole has imaginary part $\omega_d = \frac{\pi}{T_p} = \frac{\pi}{(2/3)(0.297)} = 15.87$.

- Real part is thus at $\sigma = \frac{\omega_d}{\tan(180^o - 117.13^o)} = 8.13$.

- Our desired dominant poles are thus at $s = -8.13 \pm j15.87$, which requires gain $K = 5.34$.

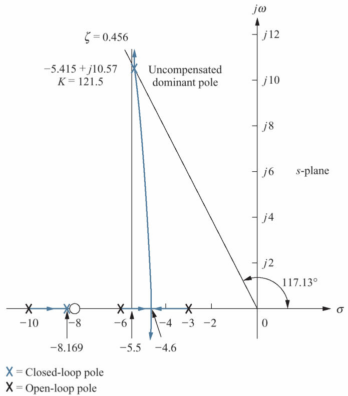


Figure 9.32

PID Controller Design Example III

- Sum of angles to $s = -8.13 + j15.87$ is currently -198.37^o.
- Zero contribution is thus $\theta_z = 198.37^0 - 180^o = 18.37^o$.
- This means we need a zero at $s = -55.92$.
- Our PD controller is thus $G_{PD} = (s + 55.92)$.

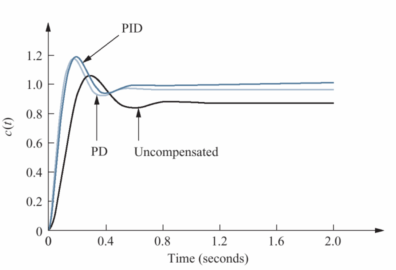


Figure 9.35

PID Controller Design Example IV

- We choose our PI controller to be $G_{PI} = \frac{(s+0.5)}{s}$
- This gives dominant poles at $s = -7.516 \pm j14.67$ at gain $K = 4.6$.
- Combining PI and PD to create PID controller, we need to determine gains K_1, K_2, and K_3.

- $G_{PID}(s) = \frac{K(s+55.92)(s+0.5)}{s} = \frac{4.6(s^2+56.42s+27.96)}{s}$
- Equating with PID controller equation, we have $K_3 = 4.6$, $\frac{K_1}{K_3} = 56.42$, and $\frac{K_2}{K_3} = 27.96$.
- Thus $K_1 = 259.5$ and $K_2 = 128.6$.

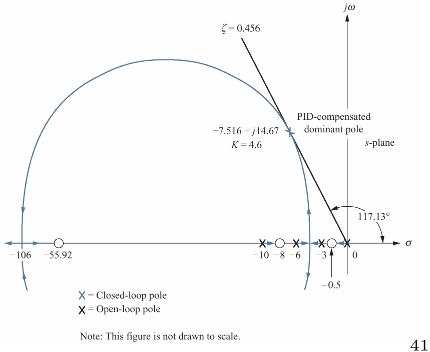


Figure 9.36

PID Controller Design Example V

- Simulation compares original, PD compensated, and PID compensated system.

- PID compensated system takes 3 seconds to get to unity.

- If need faster, redesign PD controller.

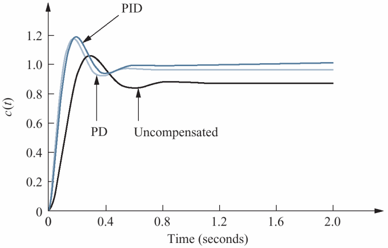


Figure 9.35

Lag-Lead Compensator Design

1. Evaluate performance of original system.
2. Design lead compensator for transient response improvement.
3. Simulate system to be sure requirements met.
4. Redesign if simulation shows they are not.
5. Evaluate steady state error performance of lead compensated system.
6. Design lag compensator for steady state error requirement.
7. Simulate system to be sure all conditions met.
8. Redesign if not met.